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THEORY OF TACHYONIC NATURE OF NEUTRINO

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Abstract

This work is aimed at attempting to establish the tachyon nature of neutrino. The basis of this theory is the neutrino's proper four-vectors and tachyon energetic right-angled triangle by means of which the movement of neutrinos is described. Emphasis is put on the occurrence of the negative value of the squared proper mass in the basic energetic equation, which could indicate possible tachyonic nature of neutrinos, and be a key factor in resolving the question of neutrino's velocity.

1. Introduction

In proposed of our theoretical work, we will focus our attention on the issues that are still open in the theoretical physics, and they are the following:

1. Processing of the experimental results ends up in a hypothesis of tachyonic neutrinos based on the negative neutrino mass squared $-m^2c^2(-m^2v^4)$. That raises the question: what in theoretical physics is the explanation of this phenomenon?

2. All of the numerous experiments, ever since neutrino was discovered, show that, neutrinos always have left-handed spin, while the antineutrinos have right-handed spin. The question is: if we consider the fact that neutrinos have mass, by which of the theoretical procedures are these physical characteristics to be explained?

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We raise these questions because of the disagreement existing between the experimental results given in the references [1-5] which indicate that neutrinos are moving at the speed close to the light, or at the speed of light, at one hand, and the indications of the tachyonic nature of neutrinos presented in references [7, 8], at the other. Of a special significance is the elaboration of some experimental results presented in the references [10, 11], which indicate the possibility of a tachyonic nature of some neutrinos, i.e., that neutrinos could be faster than light.

We shall seek the answers to the above questions through a theoretical construction of a mathematical model which makes the basis of our theory of superluminal particles in the form of a "tachyon-energy right-angled triangle", similar to the case of special theory of relativity which is founded on its "right-angled triangle of energy".

Further development of the superluminal particles theory can be followed in the structural content of this paper, divided into several parts.

Part one is denoted to the theoretical basis of neutrino's proper four-vector by means of which the physical properties of neutrinos are described.

In particular, from the written equations for the tachyonic particles one can see the occurrence of spontaneous transformation of the Lorentz factor into the tachyonic factor that becomes a key factor in defining the theory of superluminal particles.

Then the procedure for reaching the solution of the structural form of superluminal transformation matrix is offered, and further, through analysis of its properties the process of reaching the boost solution is presented, by which the final structural form of tachyonic transformation matrix is given.

Further research is meant for finding bispinor wave functions in which the main results are given in the form of spin feature of tachyonic particles and antiparticles.

2. Neutrino's Four-vectors

2.1. Proper length

In any inertial frame of reference, we define for neutrinos that the proper distance (length) S, is

$$S = \sqrt{(\Delta x^2 + \Delta y^2 + \Delta z^2) - c^2 \Delta t^2}\,;\; S^2 > 0, \tag{1}$$

where

-Δx, Δy, Δz is difference in the linear, orthogonal, spatial coordinates of the two events;

-Δt is the difference in the temporal coordinates of the two events, and c is the speed of light.

2.2. Space-like interval

There exists a reference frame such that the two events are observed to occur at the same time, but there is not a reference frame in which the two events can occur in the same spatial location.

For these space-like event pairs with the positive space-time interval $(S^2 > 0)$, the measurement of space-like separation is the proper distance, $\nabla\sigma$

$$\Delta\sigma = \sqrt{S^2} = \sqrt{\Delta r^2 - c^2\Delta t^2}. \tag{2}$$

Like the proper time of time-like intervals, the proper distance of space-like space-time intervals is a real number value.

2.3. Time-like interval

For two events separated by a time-like interval, enough time passes between them there could be a case-effect relationship between the two events. For a particle traveling through space at the speed of light or larger, any two events which occur to or by the particle must be separated. Every pair with time-like separation defines a positive squared space-time interval $(S^2 > 0)$ and may be said to occur in each other's future or past. The measure of a time-like space-time interval is described by the proper time $\Delta\tau$,

$$\Delta\tau = \sqrt{\Delta r^2 / c^2 - \Delta t^2};\ \Delta r^2 / c^2 - \Delta t^2 > 0. \tag{3}$$

The proper time interval would be measured by an observer with a clock traveling between the two events in an inertial reference frame, when the observer's path intersects each event as that event occurs. The proper time defines a real number, since the square root is positive.

$$\Delta\tau = \sqrt{\Delta r^2 / c^2 - \Delta t^2} = \sqrt{\left[(v_x\Delta t / c)^2 + (v_y\Delta t / c)^2 + (v_z\Delta t / c)^2\right] - \Delta t^2}$$

$$= \Delta t\sqrt{v^2 / c^2 - 1} = \Delta t\Gamma^{-1}; \sqrt{v_x^2 + v_y^2 + v_z^2} = v > c. \tag{4}$$

So, we have

$$\Delta t = \Gamma\Delta\tau \tag{5}$$

or a definition of the tachyon factor Γ, from here, is

$$\frac{\Delta t}{\Delta\tau} = \frac{\delta t}{\delta\tau} = \frac{dt}{d\tau} = \Gamma = \frac{1}{\sqrt{v^2 / c^2 - 1}}. \tag{6}$$

2.4. Definition of neutrino velocity and tachyonic factor

Proper velocity is one of three related derivatives in neutrino physics, where

$$w = \frac{dx}{d\tau} \tag{7}$$

is neutrino's tachyonic factor

$$\Gamma = \frac{dt}{d\tau}, \tag{8}$$

and coordinate velocity

$$v = \frac{dx}{dt} \tag{9}$$

that described an object's rate of motion.

2.5. Neutrino's four-velocity

The four coordinate function $S = X^{\alpha}(\tau)$; $\alpha = 0, 1, 2, 3$ defining a world line, a real functions of a real variable τ and can simply be differentiated in the usual calculus. The tangent vector of the world line $X^{\alpha}(\tau)$ is a four-dimensional vector, called four-velocity

$$U^{\alpha} = \frac{dX^{\alpha}}{d\tau}; \alpha = 0, 1, 2, 3. \tag{10}$$

The relationship between the time t and the coordinate time X^0 is given by

$$X^0 = ct. \tag{11}$$

Taking the derivatives with respect to the proper time τ, we find the U^0 velocity

component

$$U^0 = \frac{dX^0}{d\tau} = \frac{dX^0}{dt}\frac{dt}{d\tau} = c\Gamma. \tag{12}$$

Using the chain rule, for $\alpha = 1, 2, 3,$ we have

$$U^\alpha = \frac{dX^\alpha}{d\tau} = \frac{dX^\alpha}{dt}\frac{dt}{d\tau} = v^\alpha\Gamma;\ \alpha = 1, 2, 3. \tag{13}$$

So, we have

$$U^\alpha = \begin{pmatrix} \Gamma c \\ \Gamma v_x \\ \Gamma v_y \\ \Gamma v_z \end{pmatrix}. \tag{14}$$

2.6. Neutrino's four-momentum

For a massive particle, the four-momentum is given by the particle's invariant mass m multiplied by the particle's four-velocity

$$P^\alpha = mU^\alpha = \begin{pmatrix} P^0 \\ P^1 \\ P^2 \\ P^3 \end{pmatrix} = \begin{pmatrix} m\Gamma c \\ m\Gamma v_x \\ m\Gamma v_y \\ m\Gamma v_z \end{pmatrix}. \tag{15}$$

3. The Structure of Superluminal Matrix

Spontaneous transformation of the sign within Lorentz factor $\gamma(\beta)$ into a new form $\Gamma(\beta)$, which is characteristic of superluminal particles, is the result of a simple definition of proper time (4) made in the key formulas (1) and was obtained tachyonic factor in the form

$$\Gamma(\beta) = \left(\sqrt{\beta^2 - 1}\right)^{-1}. \tag{16}$$

This $\Gamma(\beta)$ factor, defined by the formula (16), is the superluminal factor that represents the element of superluminal matrix. To begin with there is now the task to determine the position of this factor in the superluminal matrix $T(\beta)$. To resolve the

issue, we shall depart from the simplest superluminal matrix with the boost towards the x_1-axis, and it can be written in a general form (17) as follows:

$$T(\beta) = \begin{pmatrix} T_0^0 & T_1^0 & 0 & 0 \\ T_0^1 & T_1^1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{17}$$

where the structural elements of the matrix T_0^0, T_1^0, T_0^1, T_1^1 are unknown parameters that need to be defined.

We seek solutions for these parameters assuming that the superluminal matrix creates an algebraic group belonging to the set $T \in (G\{O(3, 1), T^T g^T\})$, where g as determined by (18)

$$g = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{18}$$

represents the Minkowski space-time matrix.

The final structure of the key superluminal matrix with boost in the direction of the x_1-axis can be shown by the following formula (19):

$$T(\beta) = \begin{pmatrix} \Gamma\beta & \Gamma & 0 & 0 \\ \Gamma & \Gamma\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{19}$$

with its inverse matrix of the form (20):

$$T^{-1}(\beta) = \frac{adjT(\beta)}{\det T(\beta)} = \begin{pmatrix} \Gamma\beta & -\Gamma & 0 & 0 \\ -\Gamma & \Gamma\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{20}$$

with the determinant 1,

$$\det T = \det T^T = \det T^{-1} = \Gamma^2\beta^2 - \Gamma^2 = 1 \tag{21}$$

and with the following properties:

$$TgT = T^T gT$$

$$= \begin{pmatrix} \Gamma\beta & \Gamma & 0 & 0 \\ \Gamma & \Gamma\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \Gamma\beta & \Gamma & 0 & 0 \\ \Gamma & \Gamma\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = g, \tag{22}$$

$$T^{-1} gT^{-1} = \left(T^T\right)^{-1} gT^{-1}$$

$$= \begin{pmatrix} \Gamma\beta & -\Gamma & 0 & 0 \\ -\Gamma & \Gamma\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \Gamma\beta & -\Gamma & 0 & 0 \\ -\Gamma & \Gamma\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = g, \tag{23}$$

$$gTg$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \Gamma\beta & \Gamma & 0 & 0 \\ \Gamma & \Gamma\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = T^{-1}, \tag{24}$$

$$gT^{-1}g$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \Gamma\beta & \Gamma & 0 & 0 \\ \Gamma & \Gamma\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = T. \tag{25}$$

4. The Zero Superluminal Matrices

The zero superluminal matrices $T(0)$ and $T^{-1}(0)$ are defined if we put $\beta = 0$ from (19) and (20):

$$T(0) = \begin{pmatrix} 0 & \Gamma(0) \\ \Gamma(0) & 0 \end{pmatrix} = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}. \tag{26}$$

The Zero Superluminal Matrix $T(0)$, (26) has its inverse matrix,

$$T^{-1}(0) = \begin{pmatrix} 0 & -\Gamma(0) \\ -\Gamma(0) & 0 \end{pmatrix} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \tag{27}$$

because $\det T(0) = 1$.

The product of matrices, (26) and (27), gives the identity matrix (28):

$$T(0)T^{-1}(0) = \begin{pmatrix} 0 & \Gamma(0) \\ \Gamma(0) & 0 \end{pmatrix} \begin{pmatrix} 0 & -\Gamma(0) \\ -\Gamma(0) & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{28}$$

We point out the matrices (26) and (27) satisfy the following conditions:

$$T(0)gT(0) = g, \tag{29}$$

$$T^{-1}(0)gT^{-1}(0) = g, \tag{30}$$

$$gT(0)g = T^{-1}(0), \tag{31}$$

$$gT^{-1}(0)g = T(0) \tag{32}$$

and therefore belong to the set $T \in (G\{O(3, 1), T^T gT\})$.

Inserting appropriate matrix calculations results, (29), (30), (31), (32), we obtain relations (33), (34), (35), (36):

$$T(0)gT(0)$$

$$= \begin{pmatrix} 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = g, \tag{33}$$

$$T^{-1}(0)gT^{-1}(0)$$

$$= \begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = g, \tag{34}$$

$$gT(0)g$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = T^{-1}(0), \tag{35}$$

$$gT^{-1}(0)g$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = T(0). \tag{36}$$

5. Four-momentum Invariance and Neutrino's Velocity and Energy

For the particle along x-direction, applying the tachyonic transformation matrix,

for the S^1 inertial reference frame will be:

$$\begin{pmatrix} m\Gamma^{1,}c \\ m\Gamma^{1,}v^1 \end{pmatrix} = \begin{pmatrix} \Gamma\beta & -\Gamma \\ -\Gamma & \Gamma\beta \end{pmatrix} \begin{pmatrix} m\Gamma c \\ m\Gamma^{,}v \end{pmatrix}. \tag{37}$$

From this relation, we get invariant quantity

$$\left(m\Gamma^1 c\right)^2 - p^{1^2} = (m\Gamma c)^2 - p^2 = -m^2c^2,$$

$$\Gamma^1 = \frac{1}{\sqrt{v^{1^2}/c^2 - 1}};\ p^1 = m\Gamma^1 v^1;\ p = m\Gamma v. \tag{38}$$

Also, using Minkowski space-time metric one gets the same invariant quantity

$$\begin{pmatrix} m\Gamma c \\ m\Gamma v_x \\ m\Gamma v_y \\ m\Gamma v_z \end{pmatrix}^+ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} m\Gamma c \\ m\Gamma v_x \\ m\Gamma v_y \\ m\Gamma v_z \end{pmatrix}$$

$$= (m\Gamma c)^2 - (m\Gamma v)^2 = -m^2c^2. \tag{39}$$

It is apparent that $m\Gamma c < m\Gamma v\,(v > 0)$ and, in order to determine neutrino's speed denoted by *X*, for the neutrino's energy we may write:

$$E = m\Gamma c X, \tag{40}$$

where from here, follows: $X > v$, and we may write

$$E = pv = m\Gamma v^2. \tag{41}$$

The particle's speed we get equalizing relations (40) and (41), and one gets:

$$X = v^2/c \tag{42}$$

with its momentum

$$E/(v^2/c) = m\Gamma v^2/(v^2/c) = m\Gamma c = p_{v^2/c}. \tag{43}$$

And we conclude that such particles with the speed (42) are moving faster than light and they, apparently, represent the particles with tachyonic properties.

5.1. The expressions for momentum and energy

And now, using Minkowski space-time transformation, we get the invariant

$$\begin{pmatrix} p_{v^2/c} \\ p_x \\ p_y \\ p_z \end{pmatrix}^{+} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} p_{v^2/c} \\ p_x \\ p_y \\ p_z \end{pmatrix} = -m^2c^2 \tag{44}$$

and through tachyonic matrix in two different reference frame

$$\begin{pmatrix} p_{v^{1^2}/c}{}^{1} \\ p^1{}_x \\ p^1{}_y \\ p^1{}_z \end{pmatrix} = \begin{pmatrix} \Gamma\beta & -\Gamma & 0 & 0 \\ -\Gamma & \Gamma\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p_{v^2/c} \\ p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} \Gamma\beta p_{v^2/c} - \Gamma p_x \\ -\Gamma p_{v^2/c} + \Gamma\beta p_x \\ p_y \\ p_y \end{pmatrix}, \tag{45}$$

we have

$$p_{v^{1^2}/c}{}^{1^2} - p^{1^2} = E^{1^2} / \left(v^{1^2}/c\right)^2 - p^{1^2}$$

$$= E^2 / (v^2/c)^2 - p^2 = p_{v^2/c}{}^2 - p^2 = -m^2c^2. \tag{46}$$

And, from here, we get the equation for the particle's energy, E

$$E = (v^2/c)\sqrt{p^2 - m^2c^2} \tag{47}$$

which belongs to the tachyon neutrinos.

Then the system of equations for tachyonic neutrino becomes:

$$E = (v^2/c)\sqrt{p^2 - m^2c^2}, \tag{48}$$

$$\vec{p}\vec{v} = E. \tag{49}$$

Out of which the formulas for the momentum of particle $\vec{p}$ and its energy E:

$$\vec{p} = m\vec{v}\left(\sqrt{\beta^2 - 1}\right)^{-1}, \tag{50}$$

$$E = mv^2\left(\sqrt{\beta^2 - 1}\right)^{-1};\ \beta = v/c > 1, \tag{51}$$

where E^2 represents the square of (the length of) "hypotenuse" of the appropriate "energy right-angled triangle." In Einstein's theory of special relativity, "the legs" (catheti) of the "energy right-angled triangle" (are pc and mc^2. For the "tachyonic right-angled triangle" (47), one leg is $pv_m = p(v^2/c)$ and the other equals $imv_m c = im(v^2/c)\,c = imv^2;\ i = \sqrt{-1}$.

Thus, due to the appearance of an imaginary unity in the second member of the energy Equation (47), the energy "triangle" defined by Equation (47) is nothing else but the "tachyonic triangle." And if we now write Equation (47) in the following form

$$E^2 \,/\, v_m^2 - p^2 = E^2 \,/\, (v^2/c)^2 - p^2 = -m^2c^2, \tag{52}$$

we obtain at the end of Equation (52) a physical quantity $-m^2c^2$, known as the negative value of the product of squared mass and velocity c^2, which has for years been measured in experiments and presented as a physical quantity related to neutrinos. Thus, based on this theoretical model of superluminal particles, we can say that experiments measure neutrinos behaving as tachyonic particles. This is a reality, and this theory should be understood as a theory explaining why this tachyonic member occurs in experiments, and therefore our further research will be based on the "tachyon triangle" in order to define, after all, superluminal -tachyonic transformation matrix.

5.2. Energy-momentum invariance

The transformation matrix (19) represents the superluminal particle with energy and momentum $(E,\ \vec{P})$ in the reference frame S, and the transformation matrix (20) represents the energy and momentum $(E^1,\ \vec{P}^1)$ of the same superluminal particle observed in the reference frame S^1:

$$\begin{pmatrix} E^1 \\ \left(v^{1^2}/c\right)\vec{P}^{1,} \end{pmatrix} = \begin{pmatrix} \Gamma\beta & -\Gamma \\ -\Gamma & \Gamma\beta \end{pmatrix} \begin{pmatrix} E \\ \left(v^2/c\right)\vec{P} \end{pmatrix} = \begin{pmatrix} \Gamma\beta E - \Gamma\left(v^2/c\right)\vec{P} \\ -\Gamma E + \Gamma\beta\left(v^2/c\right)\vec{P} \end{pmatrix}. \tag{53}$$

With regard to transformation (53), the particle observed in the reference system S is defined through transformation

$$\begin{pmatrix} E \\ (v^2/c)\vec{P} \end{pmatrix} = \begin{pmatrix} \Gamma\beta & \Gamma \\ \Gamma & \Gamma\beta \end{pmatrix} \begin{pmatrix} E^1 \\ \left(v^{1^2}/c\right)\vec{P}^1 \end{pmatrix} = \begin{pmatrix} \Gamma\beta E^1 + \Gamma\left(v^{1^2}/c\right)\vec{P}^1 \\ \Gamma E^1 + \Gamma\beta\left(v^{1^2}/c\right)\vec{P}^1 \end{pmatrix}. \tag{54}$$

$E^{1^2} / \left(v^{1^2}/c\right)^2 - \vec{p}^{1^2}$; $E^2 / (v^2/c)^2 - \vec{p}^2$ represents the invariance of superluminal transformations (53) and (54)

$$E^{1^2} / \left(v^{1^2}/c\right)^2 - \vec{P}^{1^2} = E^2 / (v^2/c)^2 - \vec{P}^2 = -m^2c^2;\ v^1 > c,\ v > c. \tag{55}$$

The invariance of superluminal particle is shown with the physical value known as the negative mass square $(-m^2c^2)$ and it represents experimentally measured observable which indicates about possible tachyonic nature of neutrinos.

5.3. Four-velocity addition formula- composition of velocities

Suppose we have a second reference frame S^1 whose spatial axes and clock coincide with that of S at time zero, but it is moving at a constant velocity $\vec{v}$ with respect to S along the x-axis. Define the event to have space-time coordinate (ct, x, y, z) in system S and (ct^1, x^1, y^1, z^1) in S^1. Then tachyonic transformation, specifies that these coordinates are related to the following way:

$$\begin{pmatrix} ct^1 \\ x^1 \\ y^1 \\ z^1 \end{pmatrix} = \begin{pmatrix} \Gamma\beta & -\Gamma & 0 & 0 \\ -\Gamma & \Gamma\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \Gamma\beta ct - \Gamma x \\ -\Gamma ct + \Gamma\beta x \\ y \\ z \end{pmatrix} \tag{56}$$

which gives the following relations:

$$t^1 = \Gamma(\beta t - x/c) = \Gamma[(v/c)t - x/c],$$

$$x^1 = \Gamma(\beta x - ct) = \Gamma[(v/c)x - ct],$$

$$y^1 = y,$$

$$z^1 = z. \tag{57}$$

By similar way will be

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \Gamma\beta & \Gamma & 0 & 0 \\ \Gamma & \Gamma\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct^1 \\ x^1 \\ y^1 \\ z^1 \end{pmatrix} = \begin{pmatrix} \Gamma\beta ct^1 + \Gamma x^1 \\ \Gamma ct^1 + \Gamma\beta x^1 \\ y^1 \\ z^1 \end{pmatrix} \tag{58}$$

and

$$\begin{aligned} t &= \Gamma(\beta t^1 + x^1/c) = \Gamma\left[(v/c)t^1 + x^1/c\right], \\ x &= \Gamma(\beta x^1 + ct^1) = \Gamma\left[(v/c)x^1 + ct^1\right], \\ y &= y^1, \\ z &= z^1. \end{aligned} \tag{59}$$

If the observer in frame S measures an object moving along x-axis at velocity $u = dx/dt$, then the observer in the S^1 system, a frame of reference moving at velocity $\vec{v}$ in the x-direction with respect to S, will measure the object moving with velocity $u^1 = dx^1/dt^1$ where, from relations (57) one gets

$$u^1 = \frac{dx^1}{dt^1} = \frac{dx^1}{dt}\frac{dt}{dt^1} = \frac{vu - c^2}{v - u} \tag{60}$$

and from relations (59) one gets

$$u = \frac{dx}{dt} = \frac{dx}{dt^1}\frac{dt^1}{dt} = \frac{vu^1 + c^2}{v + u^1}. \tag{61}$$

Let we notice that in any reference frame the square of proper lengths always stay unchanged and equal to

$$\left[(x^1)^2 + (y^1)^2 + (z^1)^2\right] - (ct^1)^2 = \left[(x)^2 + (y)^2 + (z)^2\right] - (ct)^2 = S^2. \tag{62}$$

5.4. Analysis of the momentum and energy with respect to composite velocity

We take the frame S^1 moving in the x-direction at speed $\vec{v}$ relative to S. Consider a particle of rest mass m moving at $u^1 = dx^1/dt^1$ in the x^1 direction in frame S^1, and hence at $\vec{u}$ along x-axis in S, where

$$u = \frac{vu^1 + c^2}{v + u^1}. \tag{63}$$

The momentum in S is

$$p = mu\left(\sqrt{\left(u^2 / c^2\right) - 1}\right)^{-1} = mu\Gamma_u \tag{64}$$

and in S^1

$$p^1 = mu^1\left(\sqrt{\left(u^{1^2} / c^2\right) - 1}\right)^{-1} = mu^1\Gamma_u^1. \tag{65}$$

Thus

$$p = m\frac{vu^1 + c^2}{v + u^1}\frac{1}{\sqrt{\left(\frac{vu^1 + c^2}{v + u^1}\right)^2 \Big/ c^2 - 1}} \tag{66}$$

giving

$$p = \frac{mu^1(v / c) + mc}{\sqrt{v^2 / c^2 - 1}\sqrt{u^{1^2} / c^2 - 1}}$$

$$= \Gamma\Gamma_u^1 mu^1(v / c) + \Gamma\Gamma_u^1 mc = m\Gamma\Gamma_u^1 \frac{1}{c}(vu^1 + c^2) \tag{67}$$

from which we get

$$p = m\Gamma_u u = m\frac{1}{c}\Gamma\Gamma_u^1(v + u^1)u$$

$$= m\frac{1}{c}\Gamma\Gamma_u^1(v + u^1)\frac{vu^1 + c^2}{v + u^1} = m\Gamma\Gamma_u^1 \frac{1}{c}(vu^1 + c^2). \tag{68}$$

Also using the four-momentum transformation in the frame S

$$\begin{pmatrix} m\Gamma_u c \\ m\Gamma_u u \end{pmatrix} = \begin{pmatrix} \Gamma\beta & \Gamma \\ \Gamma & \Gamma\beta \end{pmatrix}\begin{pmatrix} m\Gamma_u^1 c \\ m\Gamma_u^1 u^1 \end{pmatrix} = \begin{pmatrix} \Gamma\beta m\Gamma_u^1 c + \Gamma m\Gamma_u^1 u^1 \\ \Gamma m\Gamma_u^1 c + \Gamma\beta m\Gamma_u^1 u^1 \end{pmatrix} \tag{69a}$$

and in the frame S^1

$$\begin{pmatrix} m\Gamma_u^1 c \\ m\Gamma_u^1 u^1 \end{pmatrix} = \begin{pmatrix} \Gamma\beta & -\Gamma \\ -\Gamma & \Gamma\beta \end{pmatrix} \begin{pmatrix} m\Gamma_u c \\ m\Gamma_u u \end{pmatrix} = \begin{pmatrix} \Gamma\beta m\Gamma_u c - \Gamma m\Gamma_u u \\ -\Gamma m\Gamma_u c + \Gamma\beta m\Gamma_u u \end{pmatrix}, \tag{69b}$$

we shall get

$$m\Gamma_u c = p_{u^2/c} = \Gamma\beta(m\Gamma_u^1 c) + \Gamma\Gamma_u^1 m u^1 = m\Gamma\Gamma_u^1 (v + u^1), \tag{70}$$

$$m\Gamma_u u = p = \Gamma\beta(m\Gamma_u^1 u^1) + \Gamma\Gamma_u^1 mc = m\Gamma\Gamma_u^1 \frac{1}{c}(vu^1 + c^2), \tag{71}$$

$$m\Gamma_u^1 c = p^1{}_{u^{1^2}/c} = \Gamma\beta(m\Gamma_u c) - \Gamma\Gamma_u mu = m\Gamma\Gamma_u (v + u), \tag{72}$$

$$m\Gamma_u^1 u^1 = p^1 = \Gamma\beta(m\Gamma_u u) - \Gamma\Gamma_u mc = m\Gamma\Gamma_u \frac{1}{c}(vu - c^2). \tag{73}$$

For further calculations, we will use relations (74), (75):

$$\Gamma_u = \frac{1}{\sqrt{u^2/c^2 - 1}} = \frac{1}{\sqrt{\left[(vu^1 + c^2)/(v + u^1)\right]^2/c^2 - 1}} = \frac{1}{c}\Gamma\Gamma_u^1 (v + u^1), \tag{74}$$

$$\Gamma_u^1 = \frac{1}{\sqrt{u^{1^2}/c^2 - 1}} = \frac{1}{\sqrt{\left[(vu - c^2)/(v - u)\right]^2/c^2 - 1}} = \frac{1}{c}\Gamma\Gamma_u (v - u). \tag{75}$$

The procedure for getting relations (70), (71), (72), (73) is based upon the implementation of relations (74), (75), and by direct calculations will be:

$$p_{u^2/c} = m\Gamma_u c = m\left[\frac{1}{c}\Gamma\Gamma_u^1 (v + u^1)\right]c = m\Gamma\Gamma_u^1 (v + u^1), \tag{76}$$

$$p = m\Gamma_u u = m\left[\frac{1}{c}\Gamma\Gamma_u^1 (v + u^1)\right]\frac{vu^1 + c^2}{v + u^1} = m\Gamma\Gamma_u^1 \frac{1}{c}(vu^1 + c^2), \tag{77}$$

$$p^1{}_{u^{1^2}/c} = m\Gamma_u^1 c = m\left[\frac{1}{c}\Gamma\Gamma_u (v - u)\right]c = m\Gamma\Gamma_u (v - u), \tag{78}$$

$$p^1 = m\Gamma_u^1 u^1 = m\left[\frac{1}{c}\Gamma\Gamma_u (v - u)\right]\frac{vu - c^2}{v - u} = m\Gamma\Gamma_u \frac{1}{c}(vu - c^2). \tag{79}$$

The energy in the frame S^1 is

$$E^1 = \frac{mu^{1^2}}{\sqrt{u^{1^2}/c^2 - 1}} = m\Gamma_u^1 cu^{1^2}/c = m\Gamma_u^1 u^{1^2} = m\Gamma\Gamma_u \frac{1}{c}(v - u)u^{1^2} \tag{80}$$

and in S

$$E = \frac{mu^2}{\sqrt{u^2/c^2 - 1}} = m\Gamma_u c\frac{u^2}{c} = m\Gamma_u u^2 = m\Gamma\Gamma_u^1\frac{1}{c}(v + u^1)u^2. \tag{81}$$

The four-momentum invariance in the frame S, we calculate through relation (82):

$$p^2_{u^2/c} - p^2 = (\Gamma_u mc)^2 - (\Gamma_u mu)^2 = m^2c^2\Gamma_u^2(1 - u^2/c^2)$$

$$= \left[m\frac{1}{c}\Gamma\Gamma_u^1(v + u^1)c\right]^2 - \left[m\frac{1}{c}\Gamma\Gamma_u^1(v + u^1)\frac{vu^1 - c^2}{v + u^1}\right]^2 = -m^2c^2 \tag{82}$$

and the energy-momentum invariance is

$$E^2 - p^2(u^2/c)^2 = (m\Gamma_u u^2)^2 - (m\Gamma_u u)^2(u^2/c)^2 = m^2\Gamma_u^2u^4(1 - u^2/c^2)$$

$$= \left[m\frac{1}{c}\Gamma\Gamma_u^1(v + u^1)u^2\right]^2 - \left[m\frac{1}{c}\Gamma\Gamma_u^1(vu^1 + c^2)\right]^2(u^2/c)^2 = -m^2u^4. \tag{83}$$

Also, the four-momentum invariance in the frame S^1 will be:

$$p^{1^2}_{u^{1^2}/c} - p^{1^2} = (\Gamma_u^1 mc)^2 - (\Gamma_u^1 mc^1)^2 = m^2\Gamma_u^{1^2}c^2\left(1 - u^{1^2}/c^2\right)$$

$$= \left[m\frac{1}{c}\Gamma\Gamma_u(v - u)c\right]^2 - \left[m\frac{1}{c}\Gamma\Gamma_u(v - u)\frac{vu - c^2}{v - u}\right]^2 = -m^2c^2 \tag{84}$$

and energy-momentum invariance in the frame S^1 is

$$E^{1^2} - p^{1^2}\left(u^{1^2}/c\right)^2$$

$$= m^2\Gamma_u^{1^2}u^{1^4} - m^2\Gamma_u^{1^2}u^{1^2}\left(u^{1^2}/c\right)^2 = m^2\Gamma_u^{1^2}u^{1^4}\left(1 - u^{1^2}/c^2\right)$$

$$= \left[m\frac{1}{c}\Gamma\Gamma_u(v - u)u^{1^2}\right]^2 - \left[m\frac{1}{c}\Gamma\Gamma_u(v - u)\frac{vu - c^2}{v - u}\right]^2\left(u^{1^2}/c\right)^2 = -m^2u^{1^4}. \tag{85}$$

On the basis of the calculated results given in relations (82) and (84), we can conclude that the four-momentum invariance are holding in all inertial reference

frames, until the energy-momentum relations (83) and (85) are in dependence of the particles velocities connected with appropriate reference frames.

It means that neutrino's speed is greater than the light speed in all reference frames, but it can not be as a constant speed.

6. Superluminal Matrix and Boost

The boost of superluminal particles can be determined in two ways. One is based on the product (86)

$$\Lambda(\delta)\Lambda^{-1}(\delta) = T(\delta)T^{-1}(\delta) = \begin{pmatrix} 1-\delta^2 & 0 \\ 0 & 1-\delta^2 \end{pmatrix} = I - \delta^2 I = I^2 - \delta^2 K_1^2$$

$$= (I + \delta K_1)(I - \delta K_1);$$

$$\left(K_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix};\ K_1^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I;\ I^2 = I \right) \tag{86}$$

which is based on the product of infinitesimal matrices (87) and (89), and (88) and (90):

$$\Lambda(\delta) = \begin{pmatrix} \gamma(\delta) & \beta\gamma(\delta) \\ \beta\gamma(\delta) & \gamma(\delta) \end{pmatrix} \approx \begin{pmatrix} 1 & \delta \\ \delta & 1 \end{pmatrix}, \tag{87}$$

$$T(\delta) = \begin{pmatrix} \delta\Gamma(\delta) & \Gamma(\delta) \\ \Gamma(\delta) & \delta\Gamma(\delta) \end{pmatrix} \approx \begin{pmatrix} -i\delta & -i \\ -i & -i\delta \end{pmatrix}, \tag{88}$$

$$\Lambda^{-1}(\delta) = \frac{adj\Lambda(\delta)}{\det \Lambda(\delta)} = \begin{pmatrix} \gamma(\delta) & -\delta\gamma(\delta) \\ -\delta\gamma(\delta) & \gamma(\delta) \end{pmatrix} \approx \begin{pmatrix} 1 & -\delta \\ -\delta & 1 \end{pmatrix}, \tag{89}$$

$$T^{-1}(\delta) = \frac{adjT(\delta)}{\det T(\delta)} = \begin{pmatrix} \delta\Gamma(\delta) & -\Gamma(\delta) \\ -\Gamma(\delta) & \delta\Gamma(\delta) \end{pmatrix} \approx \begin{pmatrix} -i\delta & -i \\ -i & -i\delta \end{pmatrix}, \tag{90}$$

where the parameters of infinitesimal matrices are determined by relations

$$\delta = \frac{\delta v}{c};\ \Gamma(0) = \frac{1}{\sqrt{-1}} = -i;\ \Gamma(\delta) = \frac{1}{\sqrt{\delta^2 - 1}} \approx -i,$$

$$\gamma(0) = 1;\ \gamma(\delta) = \frac{1}{\sqrt{1-\delta^2}} \approx 1. \tag{91}$$

The infinitesimal boost can also be calculated through formulas (92):

$$B(\delta) = T(\delta)T^{-1}(0) = \begin{pmatrix} -i\delta & -i \\ -i & -i\delta \end{pmatrix}\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 1 & \delta \\ \delta & 1 \end{pmatrix} = I + \delta K_1; \tag{92}$$

and inverse infinitesimal boost through formula (93):

$$B^{-1}(\delta) = \left[T(\delta)T^{-1}(0)\right]^{-1} = T(0)T^{-1}(\delta) = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}\begin{pmatrix} -i\delta & i \\ i & -i\delta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -\delta \\ -\delta & 1 \end{pmatrix} = I - \delta K_1. \tag{93}$$

And product of (92) and (93) will be:

$$B(\delta)B^{-1}(\delta) = T(\delta)T^{-1}(0)\left[T(\delta)T^{-1}(0)\right]^{-1} = T(\delta)\left[T^{-1}(0)T(0)\right]T^{-1}(\delta)$$

$$= T(\delta)T^{-1}(\delta) = I^2 - \delta^2 K_1^2. \tag{94}$$

For arbitrarily small value $\beta = v / c = \delta$, we assign the value for n by providing that for consecutive differentials δ, we take that n tends towards infinity, while the product $n\delta$ becomes fixed at a specific value, i.e., $n\delta = \xi$. Then, based on (19), the resultant superluminal transformation matrix can be written in the form of expression (95):

$$T(\beta) = \begin{pmatrix} \Gamma\beta & \Gamma \\ \Gamma & \Gamma\beta \end{pmatrix} = \lim_{n\to\infty}\left(I + \frac{\xi}{n}K_1\right)^n = e^{\xi K_1}; \tag{95}$$

$$T^{-1}(\beta) = \begin{pmatrix} \Gamma\beta & -\Gamma \\ -\Gamma & \Gamma\beta \end{pmatrix} = \lim_{n\to\infty}\left(I - \frac{\xi}{n}K_1\right)^n = e^{-\xi K_1}. \tag{96}$$

Now, with the development into the series of exponential superluminal matrix functions (96), we obtain the matrix structure of the superluminal particle boost in the form (97)

$$T(\beta) = \begin{pmatrix} \Gamma(\beta)\beta & \Gamma(\beta) \\ \Gamma(\beta) & \Gamma(\beta)\beta \end{pmatrix} = e^{\xi K_1} = \sum_{n=0}^{\infty}\frac{(\xi K_1)^n}{n!} = \sum_{n=0}^{\infty}\frac{\xi^{2n}}{(2n)!}I + \sum_{n=0}^{\infty}\frac{\xi^{2n+1}}{(2n+1)!}K_1$$

$$= \cosh\xi\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \sinh\xi\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \cosh\xi & \sinh\xi \\ \sinh\xi & \cosh\xi \end{pmatrix}. \tag{97}$$

And inverse matrix of superluminal boost will be:

$$T^{-1}(\beta) = \begin{pmatrix} \Gamma(\beta)\beta & -\Gamma(\beta) \\ -\Gamma(\beta) & \Gamma(\beta)\beta \end{pmatrix} = e^{-\xi K_1}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{(\xi K_1)^n}{n!} = \sum_{n=0}^{\infty} \frac{\xi^{2n}}{(2n)!} I + \sum_{n=0}^{\infty} (-1)^{2n+1} \frac{\xi^{2n+1}}{(2n+1)!} K_1$$

$$= \cosh \xi \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \sinh \xi \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \cosh \xi & -\sinh \xi \\ -\sinh \xi & \cosh \xi \end{pmatrix},$$

$$(K_1)^{2n} = I; \ (K_1)^{2n+1} = K_1. \tag{98}$$

If we compare the corresponding matrix elements in the formulas (97) and (98), we get:

$$\Gamma(\beta)\beta = \cosh \xi; \ \Gamma(\beta) = \sinh \xi,$$

$$\beta = \coth \xi. \tag{99}$$

By combining two consecutive superluminal boosts $\beta_1 = v_1 / c = \coth \xi_1$; $\beta_2 = v_2 / c = \coth \xi_2$, we get the resulting boost

$$\beta = \frac{v}{c} = \coth(\xi_1 + \xi_2) = \frac{1 + \coth \xi_1 \coth \xi_2}{\coth \xi_1 + \coth \xi_2} = \frac{1 + \beta_1 \beta_2}{\beta_1 + \beta_2} \tag{100}$$

and the explicit value of the resultant velocity v becomes:

$$v = (c^2 + v_1 v_2) / (v_1 + v_2). \tag{101}$$

7. Bispinor Matrix Generator

Based on the functional similarity of the elements of Lorentz and superluminal matrices, we introduce the bispinor of superluminal matrix generator in the form of transformation matrices

$$S = e^{\frac{1}{2}\vec{\xi}_1 \vec{K}_1} = \begin{pmatrix} \cosh(\xi_1 / 2) & \sinh(\xi_1 / 2) \\ \sinh(\xi_1 / 2) & \cosh(\xi_1 / 2) \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{(\Gamma\beta_1 + 1)/2} & \sqrt{(\Gamma\beta_1 - 1)/2} \\ \sqrt{(\Gamma\beta_1 - 1)/2} & \sqrt{(\Gamma\beta_1 + 1)/2} \end{pmatrix}. \tag{102}$$

The wave eigenfunctions of superluminal particle for positive energy with the momentum $\vec{p}$ in the laboratory reference frame is $\Psi(\vec{p}, +|x^{\mu})$ and $\Psi(0, +|x^{\mu})$ is the wave eigenfunction in the rest reference frame, are linked by transformation relation

$$\Psi(\vec{p}, +|x^{\mu}) = S\Psi(0, +|x^{\mu}). \tag{103}$$

And these two eigenfunctions $\Psi(\vec{p}, -|x^{\mu})$, and $\Psi(0, -|x^{\mu})$ are linked by transformational relations (104).

$$\Psi(\vec{p}, -|x^{\mu}) = S\Psi(0, -|x^{\mu}). \tag{104}$$

We shall also apply the following transformational relations by means of which to define existence of the particle:

$$\Psi^{+}(\vec{p}, \pm|x^{\mu})\gamma^{0}\Psi(\vec{p}, \pm|x^{\mu})$$

$$= \left[S\Psi(0, \pm|x^{\mu})\right]^{+}\gamma^{0}S\Psi(0, \pm|x^{\mu}) = \Psi^{+}(0, \pm|x^{\mu})\gamma^{0}\Psi(0, \pm|x^{\mu}) \tag{105}$$

while for determining the parameters of the wave functions, we use the following normalization conditions:

$$\Psi^{+}(\vec{p}, \pm|x^{\mu})\gamma^{0}\Psi(\vec{p}, \pm|x^{\mu}) = \pm 1, \tag{106}$$

$$\Psi^{+}(0, \pm|x^{\mu})\gamma^{0}\Psi(0, \pm|x^{\mu}) = \pm 1 \tag{107}$$

and thus the invariant properties of superluminal particles are maintained.

8. Schrödinger Theory of Bispinor Wave Functions

We shall ground our analysis of superluminal particles properties on the energetic relation

$$E = (v^{2}/c)\sqrt{p^{2} - m^{2}c^{2}} \tag{108}$$

and the appropriate Hamiltonian operator (109)

$$\hat{H}_{v^{2}/c} = -(v^{2}/c)(\vec{\alpha}\vec{p}) + mv^{2}\hat{\beta}_{T}$$

$$= -(v^{2}/c)\begin{pmatrix} 0 & \vec{\sigma}_{i}\vec{p}_{i} \\ \vec{\sigma}_{i}\vec{p}_{i} & 0 \end{pmatrix} + mv^{2}\begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \tag{109}$$

where relations between matrices are given below.

$$\beta_T = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}; \beta_T^2 = -I; (\vec{\alpha}\vec{p})\hat{\beta}_T + \hat{\beta}_T(\vec{\alpha}\vec{p}) = \begin{pmatrix} 0 & \vec{\sigma}\vec{p} \\ \vec{\sigma}\vec{p} & 0 \end{pmatrix}\begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$$

$$+\begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}\begin{pmatrix} 0 & \vec{\sigma}\vec{p} \\ \vec{\sigma}\vec{p} & 0 \end{pmatrix} = 0; \vec{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \vec{\sigma}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \vec{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (110)$$

Now, we introduce bispinor functions (111)

$$\Psi\left(x^{\mu}\right) = \begin{pmatrix} \phi(x^{\mu}) \\ \chi(x^{\mu}) \end{pmatrix} = \begin{pmatrix} \phi_0 \\ \chi_0 \end{pmatrix} \exp\left[\frac{i}{\hbar}(\vec{p}\vec{r} - \varepsilon t)\right] \quad (111)$$

and apply the quantum mechanics procedure by means of the Schröinger Equation (112),

$$i\hbar\frac{\partial\Psi(x^{\mu})}{\partial t} = \hat{H}_{v^2/c}\Psi(x^{\mu}) = \left[-(v^2/c)(\vec{\alpha}\vec{p}) + mv^2\hat{\beta}_T\right]\begin{pmatrix} \phi_0 \\ \chi_0 \end{pmatrix}$$

$$= -(v^2/c)\begin{pmatrix} 0 & \vec{\sigma}\vec{p} \\ \vec{\sigma}\vec{p} & 0 \end{pmatrix}\begin{pmatrix} \phi_0 \\ \chi_0 \end{pmatrix} + mv^2\begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}\begin{pmatrix} \phi_0 \\ \chi_0 \end{pmatrix} = \varepsilon\begin{pmatrix} \phi_0 \\ \chi_0 \end{pmatrix}, \quad (112)$$

where

$$\varepsilon = \pm(v^2/c)\sqrt{p^2 - m^2c^2} = \pm E(\vec{p}) \quad (113)$$

to determine them in an explicit form.

Our goal is to obtain two-dimensional spin states ϕ_0 (114) and χ_0 (115), and the superluminal wave function for positive energy $\Psi(\vec{p}, +|x^{\mu})$ (116) and the superluminal wave function for negative energy $\Psi(\vec{p}, -|x^{\mu})$ (117).

$$\phi_0 = -\frac{(v^2/c)\vec{\sigma}\vec{p} - mv^2 I}{\varepsilon}\chi_0, \quad (114)$$

$$\chi_0 = -\frac{(v^2/c)\vec{\sigma}\vec{p} + mv^2 I}{\varepsilon}\phi_0, \quad (115)$$

$$\psi(\vec{p}, +|x^{\mu}) = N_+(\vec{p})\left(-\frac{(v^2/c)\overset{u}{\vec{\sigma}\vec{p}} + mv^2}{E(\vec{p})}u\right)e^{\frac{i}{\hbar}(\vec{p}\vec{r} - E(\vec{p})t)}$$

$$= N_+(\vec{p})\left(-\frac{\overset{u}{\vec{\sigma}\vec{p}+mc}}{\sqrt{p^2-m^2c^2}}u\right)e^{\frac{i}{\hbar}(\vec{p}\vec{r}-E(\vec{p})t)};\ u^+u=1;\ \varepsilon=+E(\vec{p}), \tag{116}$$

$$\psi(\vec{p},-|x^\mu)=N_-(\vec{p})\left(-\frac{(v^2/c)\vec{\sigma}\vec{p}-mv^2}{\underset{u}{E(\vec{p})}}u\right)e^{\frac{i}{\hbar}(\vec{p}\vec{r}+E(\vec{p})t)}$$

$$= N_-(\vec{p})\left(\frac{\vec{\sigma}\vec{p}-mc}{\underset{u}{\sqrt{p^2-m^2c^2}}}u\right)e^{\frac{i}{\hbar}(\vec{p}\vec{r}+E(\vec{p})t)};\ u^+u=1;\ \varepsilon=-E(\vec{p}). \tag{117}$$

Now we shall consider and analyze the two possible spin projections in relation to the movement of superluminal particles.

9. Analysis of the Bispinor Eigenfunction in the Conditions $\vec{\sigma}\vec{p}=p$

The superluminal wave eigenfunction $\psi(\vec{p},+|x^\mu)$ (118) is used for obtaining the normalization constant $N_+(\vec{p})$ based on the terms of normalization (119), while for the anti-superluminal particle wave eigenfunction, we use its wave function (120), and $N_-(\vec{p})$ is its normalization constant obtained on condition (121).

$$\psi(\vec{p},+|x^\mu)=N_+(\vec{p})\left(-\frac{\overset{u}{p+mc}}{\sqrt{p^2-m^2c^2}}u\right)e^{\frac{1}{\hbar}(\vec{p}\vec{r}-E(\vec{p})t)}$$

$$= N_+(\vec{p})\left(-\sqrt{\frac{\overset{u}{p+mc}}{p-mc}}u\right)e^{\frac{1}{\hbar}(\vec{p}\vec{r}-E(\vec{p})t)};\ u^+u=1;\ \varepsilon=+E(\vec{p}), \tag{118}$$

$$\psi^+(\vec{p},+|x^\mu)\gamma^0\psi(\vec{p},+|x^\mu)=1. \tag{119}$$

$$\psi(\vec{p},-|x^\mu)=N_-(\vec{p})\left(\frac{p-mc}{\underset{u}{\sqrt{p^2-m^2c^2}}}u\right)e^{\frac{i}{\hbar}(\vec{p}\vec{r}-E(\vec{p})t)}$$

$$= N_-(\vec{p})\left(\sqrt{\frac{p-mc}{p+mc}}\,\underset{u}{u}\right)e^{\frac{i}{\hbar}(\vec{p}\vec{r}-E(\vec{p})t)};\ u^+u = 1;\ \varepsilon = -E(\vec{p}), \tag{120}$$

$$\psi^+(\vec{p}, -|x^\mu)\gamma^0\psi(\vec{p}, -|x^\mu) = -1. \tag{121}$$

Now we use the normalization condition (119) and obtain the explicit form (122) as an imaginary value for the $N_+(\vec{p})$ constant. Based on this, we conclude that such superluminal particle does not exist in nature

$$N_+^2(\vec{p})\left(-\sqrt{\frac{p+mc}{p-mc}}\,\underset{u}{u}\right)^+\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\left(-\sqrt{\frac{p+mc}{p-mc}}\,\underset{u}{u}\right) = N_+^2(\vec{p})\left(1-\frac{p+mc}{p-mc}\right)$$

$$= N_+^2(\vec{p})\left(\frac{-2mc}{p-mc}\right) = 1. \tag{122}$$

Now we use normalization conditions for a superluminal antiparticle (121) and get the explicit value for the normalization constant $N_-(\vec{p})$ (124) and the bispinor field $\psi(\vec{p}, -|x^\mu)$ (125) as an eigenvector of the spin operator with the eigenvalue of $+1/2$

$$\psi^+(\vec{p}, -|x^\mu)\gamma^0\psi(\vec{p}, -|x^\mu) = N_-^2(\vec{p})\left(\sqrt{\frac{p-mc}{p+mc}}\,\underset{u}{u}\right)^+\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\left(\sqrt{\frac{p-mc}{p+mc}}\,\underset{u}{u}\right) = -1,$$

$$u^+u = 1;\ \varepsilon = -E(\vec{p}), \tag{123}$$

$$N_-(\vec{p}) = \sqrt{\frac{p+mc}{2mc}} = \cosh(\xi/2), \tag{124}$$

$$\psi\left(p\hat{e}_3, -\left|x^\mu, +\frac{1}{2}\right|\vec{r}, t\right) = \cosh(\xi_3/2)\begin{pmatrix}\tanh(\xi_3/2)u\\ u\end{pmatrix}e^{\frac{i}{\hbar}\left(px^3+E(\vec{p})t\right)}$$

$$= \begin{pmatrix}\sinh(\xi_3/2)\begin{pmatrix}1\\0\end{pmatrix}\\ \cosh(\xi_3/2)\begin{pmatrix}1\\0\end{pmatrix}\end{pmatrix}e^{\frac{i}{\hbar}\left(px^3+E(\vec{p})t\right)};\ u^+u = 1;\ \varepsilon = -E(\vec{p}). \tag{125}$$

Using the formula (104), we obtain the formula (125), which means confirmation of

the existence of such an antiparticle in the nature, and thereon we conclude that the superluminal antiparticle (antineutrino) does exist in the nature and that it has the right-handed spin, which is consistent with experimental measurements of the antineutrino spin.

10. Analysis of the Bispinor Eigenfunction in Conditions $\vec{\sigma}\vec{p} = -p$

In this case, for the positive energy of the superluminal particle (126), and based on the normalization conditions (127), we obtain the bispinor field (128) $\psi(\vec{p}, +|x^{\mu})$ as an eigenvector of the spin operator with the eigenvalue of $-1/2$, and through checking (103), we conclude that the superluminal particle (neutrino) does exist in the nature and to have the left-handed spin, which is in agreement with experimental measurements of the neutrino spin.

$$\psi(\vec{p}, +|x^{\mu}) = N_{+}(\vec{p})\begin{pmatrix} u \\ \sqrt{\dfrac{p-mc}{p+mc}}\, u \end{pmatrix} e^{\frac{i}{\hbar}(\vec{p}\vec{r}-E(\vec{p})t)};\ u^{+}u = 1;\ \varepsilon = +E(\vec{p}), \quad (126)$$

$$\psi^{+}(\vec{p}, +|x^{\mu})\gamma^{0}\psi(\vec{p}, +x^{\mu})$$

$$= N_{+}^{2}\begin{pmatrix} u \\ \sqrt{\dfrac{p-mc}{p+mc}}\, u \end{pmatrix}^{+}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} u \\ \sqrt{\dfrac{p-mc}{p+mc}}\, u \end{pmatrix} = 1;\ u^{+}u = 1;\ \varepsilon = +E(\vec{p}) \quad (127)$$

$$\psi\left(p\hat{e}_3, +\left|x^{\mu}, -\frac{1}{2}\right|r, t\right) = \begin{pmatrix} \cosh(\xi_3/2)\begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ \sinh(\xi_3/2)\begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix} e^{\frac{i}{\hbar}\left(px^3-E(\vec{p})t\right)},$$

$$u^{+}u = 1;\ \varepsilon = +E(\vec{p}). \quad (128)$$

Also, by a similar procedure, we come to the conclusion that the superluminal antiparticle (antineutrino) with the left-handed spin does not exist in the nature, which is also confirmed by experiments.

11. The Velocity of Superluminal Particle

Now we examine the superluminal particles velocity that have the energy value

$E = \pm(v^2 / c)\sqrt{p^2 - m^2c^2}$ and if we put the Hamiltonian operator, $\widehat{H}_{v^2/c}$ in the Poisson bracket, we shall obtain the time evolution operator as the superluminal particle velocity.

$$\frac{d\widehat{x}_i}{dt} = \frac{1}{i\hbar}\left[\widehat{x}_i,\, \widehat{H}_{v^2/c}\right] = \frac{1}{i\hbar}\left[\widehat{x}_i,\, -\frac{v^2}{c}(\vec{\alpha}_i\vec{p}_i) + mv^2\vec{\beta}_T\right]$$

$$= \frac{1}{i\hbar}\frac{v^2}{c}\begin{pmatrix} 0 & \vec{\sigma}_i\widehat{p}_i\widehat{x}_i \\ \vec{\sigma}_i\widehat{p}_i\widehat{x}_i & 0 \end{pmatrix};\ i = 1,\ 2,\ 3. \tag{129}$$

In the following mathematical research, we have two different cases.

11.1. The key-point in the bispinor eigen-function analysis is the helicity spin condition: $\vec{\sigma}\vec{p} = p$

Under the helicity spin condition, $\vec{\sigma}\vec{p} = p$ the superluminal antiparticle (antineutrino) with right handed spin exists, and, using the speed operator given in the formula (129), we have:

$$\frac{d\widehat{x}_i}{dt} = \frac{v^2}{c}\frac{1}{i\hbar}\begin{pmatrix} 0 & \widehat{p}_i\widehat{x}_i \\ \widehat{p}_i\widehat{x}_i & 0 \end{pmatrix}$$

$$= \frac{v^2}{c}\frac{1}{i\hbar}\begin{pmatrix} 0 & -i\hbar\dfrac{\partial}{\partial x_i}\widehat{x}_i \\ -i\hbar\dfrac{\partial}{\partial x_i}\widehat{x}_i & 0 \end{pmatrix} = -\frac{v^2}{c}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{130}$$

Because of the fact that the antiparticles (antineutrinos) are moving in the opposite direction with respect to the particles (neutrinos), we may conclude that this result is proper. The antiparticles (antineutrinos) possess the right-handed spin whose projection is collinear with its momentum.

The antiparticles (antineutrinos) have the negative energy value and are moving in the opposite direction to the particles (neutrinos) that have the positive energy value.

11.2. The key-point in the analysis is the helicity spin condition: $\vec{\sigma}\vec{p} = -p$

Under the helicity spin condition, $\vec{\sigma}\vec{p} = -p$, we can say that superluminal

particles (neutrinos) with the left-handed spin exist with the speed operator given in the formula (131)

$$\frac{d\hat{x}_i}{dt} = \frac{v^2}{c}\frac{1}{i\hbar}\begin{pmatrix} 0 & -\hat{p}_i\hat{x}_i \\ -\hat{p}_i\hat{x}_i & 0 \end{pmatrix}$$

$$= \frac{v^2}{c}\frac{1}{i\hbar}\begin{pmatrix} 0 & i\hbar\frac{\partial}{\partial x_i}\hat{x}_i \\ i\hbar\frac{\partial}{\partial x_i}\hat{x}_i & 0 \end{pmatrix} = \frac{v^2}{c}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{131}$$

which gives the proper result; because particles (neutrinos) have the positive energy and they are moving along $+x_i$ axis.

11.3. The dependence between neutrino's velocity and energy

In order to define maximum neutrino's velocity, we shall analyze extremum of energy E

$$\frac{\partial E}{\partial u} = \frac{\partial}{\partial u}\left[mu^2\left(\sqrt{u^2/c^2-1}\right)^{-1}\right] = \frac{\partial(pu)}{\partial u} = \frac{\partial p}{\partial u}u + p$$

$$= \frac{\partial}{\partial u}\left[mu\left(\sqrt{u^2/c^2-1}\right)^{-1}\right]u + mu\left(\sqrt{u^2/c^2-1}\right)^{-1} = 0 \tag{132}$$

from which we get

$$u = \sqrt{2}c \tag{133}$$

in other words, the maximum of measured speed, in accordance with (130) and (131) will be:

$$(u^2/c)_{\max} = 2c \tag{134}$$

with the momentum of

$$p_{u^2/c} = m\Gamma_u c = m\frac{1}{\sqrt{2c/c-1}}c = mc \tag{135}$$

and with minimal energy of

$$E_{\min} = m\Gamma_u u^2 = m\frac{1}{\sqrt{(u^2/c)/c-1}}u^2$$

$$= \frac{m(2c^2)}{\sqrt{2c/c-1}} = p_{u^2/c}\frac{u^2}{c} = mc(2c) = 2mc^2. \tag{136}$$

If we take $u = c(1+\delta)$ in Equation (81), we shall get

$$(1+\delta)^4 - \frac{E^2}{m^2c^4}(1+\delta)^2 + \frac{E^2}{m^2c^4} = 0. \tag{137}$$

From Equation (137), there are two solutions:

$$(1+\delta)^2 = \frac{E^2}{2m^2c^4}\left[1 \pm \sqrt{1 - \frac{4m^2c^4}{E^2}}\right]. \tag{138}$$

Of these two solutions, we take in consideration the solution with the minus sign:

$$(1+\delta)^2 = \frac{E^2}{2m^2c^4}\left[1 - \sqrt{1 - \frac{4m^2c^4}{E^2}}\right] \tag{139}$$

and under the conditions $4m^2c^4 << E^2$ relation (139) becomes:

$$(1+\delta)^2 = \frac{E^2}{2m^2c^4}\left[1 - \sum_{k=0}^{\infty}\binom{1/2}{k}\left(-\frac{4m^2c^4}{E^2}\right)^k\right]$$

$$\approx \frac{E^2}{2m^2c^4}\left[1 - \left(1 - \frac{2m^2c^4}{E^2} - \frac{2m^4c^8}{E^4}\right)\right] = 1 + \frac{m^2c^4}{E^2}. \tag{140}$$

When neutrino's energy tends towards infinity will be

$$\lim_{E\to\infty}\frac{m^2c^4}{E^2} \to 0 \tag{141}$$

and then neutrino's speed tends towards the light speed:

$$\frac{u^2}{c^2} = \frac{c^2(1+\delta)^2}{c^2} \to 1. \tag{142}$$

Or in explicit form we have:

$$\frac{u^2}{c} \to c. \tag{143}$$

On the basis of Equation (140) one obtains:

$$\delta = -1 + \sqrt{1 + \frac{m^2c^4}{E^2}} \approx \frac{m^2c^4}{2E^2}\left(1 - \frac{m^2c^4}{4E^2}\right) \tag{144}$$

and from this relation follows: $\lim \delta_{E \to \infty} = 0$.

For greater energy, neutrinos slow down, till their energy tends towards the infinite amount, their speed tends towards the speed of light.

12. Conclusion

Based on the proposed model of the mathematical theory of the superluminal particle physics, we underline the major results obtained on the basis of this theory, and they are as follows:

1. There are two types of superluminal particles:

Type one of the superluminal particles (neutrinos) follow on the solution with positive energy and they have the left-handed spin.

Type two of the superluminal antiparticles (antineutrinos) follow on the solution with negative energy and they have the right-handed spin.

2. Taking into account the experimental inferences of the results related to the physically measured negative eigenvalue of the anti (neutrino) squared mass, which occurs in the theory proposed as an invariant physical quantity, we can conclude:

2a. Neutrinos and antineutrinos, whose origin is from some neutrino sources, for example, on the basis of those data and supposedly for tachyonic hypothesis based on the tritium beta decay, Supernova 1987A data and others, are the particles that could belong to the tachyonic particle type.

2b. In accordance with 2a., neutrinos and antineutrinos could be the particles with the properties of superluminal particles moving faster than light, and they were mathematically proven by this theory.

3. By this theory has been predicted of being neutrino's rest (proper) masses which experimentally has been observed [14, 15].

4. The dependence between energy and the neutrino's speed has been shown, and the maximum neutrino's speed is linked with its minimal energy. Considering the

left part of the function (51) (from the minimum of this function in direction to the speed of the light) we can conclude: for greater energy, neutrinos slow down, till their energy tends towards the infinite amount, their speed tends towards the speed of light.

The processes understanding of the deeper origin in the nature of neutrinos are unknown, especially of the being dark matter and dark energy whose investigation is at the very beginning [13].

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