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THE WIENER INDEX AND HOSOYA POLYNOMIAL OF A CLASS OF JAHANGIR GRAPHS $J_{3,m}$

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Abstract

In this paper, the Wiener Index $W(G)=\sum_{\{v,u\}\in V(G)} d(v,u)$ and Hosoya polynomial $H(G,x)=\sum_{\{v,u\}\in V(G)} x^{d(v,u)}$ of a class of Jahangir graphs $J_{3,m}$ with exactly $3m+1$ vertices and $4m$ edges are computed.

1. Introduction

Suppose G is a connected graph and $x, y \in V(G)$, where $V(G)$ denotes the set of all vertices in G. The distance $d(x, y)$ between x and y is defined as the length of a minimal path connecting x and y. The maximum distance between two vertices of G is called the diameter of G, denoted by $d(G)$. The Wiener index of a connected graph $G, W(G)$ is defined as the summation of all distances between all pairs of vertices of G. The Wiener index is introduced by Harold Wiener [1] in 1947 and is

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equal to

$$W(G) = \frac{1}{2} \sum_{v \in V(G)} \sum_{u \in V(G)} d(v, u).$$

Other properties and applications of Wiener index can be found in [2-15].

The polynomial $H(G, x) = \frac{1}{2} \sum_{v \in V(G)} \sum_{u \in V(G)} x^{d(v,u)}$ is called the Hosoya polynomial of G. The name is used in honour of Haruo Hosoya, who discovered a new formula for the Wiener index in terms of graph distance [16]. We refer the interested readers to papers [17-23] for more information on this topic and the Hosoya polynomial.

2. Main Results

In this section, we compute the Wiener index and Hosoya polynomial for Jahangir graphs. Suppose $J_{3,m}$ is Jahangir graph $\forall m \geq 3$, with exactly $3m + 1$ vertices and $4m$ edges as shown in Figure 1 [24, 25]. $\forall m \geq 3$, Jahangir graph is a connected graph consisting of a cycle C_{3m} with one additional vertex (Center vertex c) such that c is adjacent to m vertices of C_{3m} at distance 3 to each other on C_{3m}.

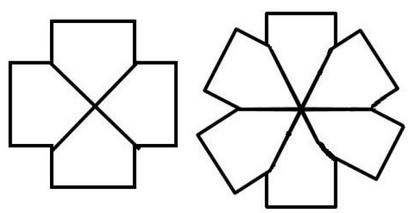

Figure 1. Some examples of Jahangir graphs $J_{3,4}$ and $J_{3,6}$.

Theorem 1. *Let* $J_{3,m}$ *be Jahangir graphs* $\forall m \geq 3$. *Then*:

The Wiener index of $J_{3,m}$ *is equal to*

$$W(J_{3,m}) = 15m^2 - 13m.$$

The Hosoya polynomial of $J_{3,m}$ *is equal to*

$$H(J_{3,m}) = 4mx^1 + \frac{1}{2}m(m+9)x^2 + 2m(m-1)x^3 + m(2m-5)x^4.$$

Proof. Suppose $J_{3,m}$ denotes Jahangir graph for all positive integer number $m \geq 3.$ From the definition of Jahangir graph $J_{3,m},$ one can see that the size of vertex set $V(J_{3,m})$ is equal to $2m + m + 1 = 3m + 1$ such that there are $2m$ vertices of Jahangir graph $J_{3,m}$ with degree two and m vertices of $J_{3,m}$ with degree three and Center vertex c has degree m. These imply that the size of edge set $E(J_{3,m})$ be

$$|E(J_{3,m})| = \frac{2 \times 2m + 3 \times m + m \times 1}{2} = 4m.$$

To compute the Wiener index and Hosoya polynomial of $J_{3,m},$ we first introduce some notions, which are useful to aims in this paper. The number of unordered pairs of vertices x and y of $J_{3,m}$ such that distance $d(x, y) = k$ is denoted by $d(J_{3,m}, k).$ Obviously $1 \leq k \leq d(J_{3,m}).$ Thus we redefine the Hosoya polynomial and Wiener index as follows:

$$H(J_{3,m}, x) = \sum_{k=1}^{d(J_{3,m})} d(J_{3,m}, k)x^k$$

and

$$W(J_{3,m}) = \sum_{k=1}^{d(J_{3,m})} d(J_{3,m}, k) \times k.$$

We divide the vertex set $V(J_{3,m})$ of Jahangir graph into several partitions on based d_v and denote by V_2, V_3 and V_m such that $V_k = \{v \in V(J_{3,m}) | d_v = k\}.$ Thus

$$V_2 = \{v \in V(J_{3,m}) | d_v = 2\} \to |V_2| = 2m,$$

$$V_3 = \{v \in V(J_{3,m}) | d_v = 3\} \to |V_2| = m,$$

$$V_m = \{c \in V(J_{3,m}) | d_c = 3\} \to |V_m| = 1.$$

From the definition of the Hosoya polynomial, it s easy to see that the first sentence of $H(J_{3,m}, x), d(J_{3,m}, 1)$ is equal to the number of edges of $J_{3,m}(4m).$

By definition of Jahangir graph $J_{3,m},$ one can see that $2m$ 2-edges paths start from the vertex c and for every vertex v of $V_2,$ there are three 2-edges paths and

$(2+(m-1))$ 2-edges paths start from a vertex u of V_3. So $d(J_{3,m}, 2) = \frac{1}{2}[2m+3(2m)+m(2+(m-1))] = \frac{1}{2}(m^2+9m)$.

From Figure 1, we see that there is not any 3-edges path with c. But there are $(2+2(m-3))$ 3-edges paths start from a vertex u of V_3 and also there are $(2+(m-2))$ 3-edges paths start from a member of V_2. Thus $d(J_{3,m}, 3) = \frac{1}{2}[0+2m(2+(m-2))+m(2+2(m-3))] = 2m(m-1)$.

From the structure of $J_{3,m}$ in Figure 1, we see that the diameter of Jahangir graph $D(J_{3,m})$ is equal to 4 and is between two vertices of V_2. The format of a 4-edges path of $J_{3,m}$ is denoted by $v_2v_3cv_2\text{'}v_3\text{'}$, where $c \in C$, $v_2, v_2\text{'} \in V_2$ and $v_3, v_3\text{'} \in V_3$. The number of these 4-edges paths is $m(1+2(m-3)) = m(2m-5)$ $(= d\ (J_{3,m}, 4))$.

Thus by these results, the Hosoya polynomial of Jahangir graph $J_{3,m}$ is equal to

$$\begin{aligned} H(J_{3,m}, x) &= \sum d(J_{3,m}, i)x^i \\ &= 4m\,x^1 + \frac{1}{2}m(m+9)x^2 + 2m(m-1)x^3 + m(2m-5)x^4. \end{aligned}$$

It is obvious that for Jahangir graph $J_{3,m}$, $H(J_{3,m}, 1) = \sum_{i=1}^{d(J_{3,m})} d(J_{3,m}, i) = \binom{3m+1}{2} = \frac{3m(3m+1)}{2}$. On the other hand, we have following computations for the Wiener index of Jahangir graph $J_{3,m}$ as:

$$\begin{aligned} W(J_{3,m}) &= \sum d(J_{3,m}, i) \times i \\ &= 1 \times 4m + 2 \times \frac{1}{2}m(m+9) + 3 \times 2m(m-1) + 4 \times m(2m-5) \\ &= 4m + m^2 + 9m + 6m^2 - 6m + 8m^2 - 20m \\ &= 15m^2 - 13m. \end{aligned}$$

And these complete the proof of Theorem 1.

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