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STUDIES OF KLEIN-GORDON PARTICLES WITH DEFORMED GENERALIZED HULTHEN POTENTIAL

AKANINYENE DANIEL ANTIA[*] and EFFIONG EKPENYONG EYIBIO

Theoretical Physics Group
Department of Physics
University of Uyo
Uyo, Nigeria
e-mail: antiacauchy@yahoo.com

Abstract

We have presented the approximate solutions of Klein-Gordon equation with Deformed Generalized Hulthen potential for equal scalar and vector fields using generalized parametric NU method with an improved approximation scheme. The energy eigenvalues and the corresponding wave functions expressed in terms of the Jacobi polynomials have been obtained. The numerical values of our results have also been computed and gives expected negative values which indicate the bound state nature of this system and also suggest usefulness for other physical systems.

1. Introduction

The exact solutions of the non-relativistic and relativistic equations with the central potential play an important role in quantum mechanics [1-4]. For example, the exact solutions of the Klein-Gordon equation for a hydrogen atom and a harmonic oscillator [2, 4] have been investigated in recent times [5]. In fact, exact solution of a quantum system is significant in physics. Solving the non-relativistic and relativistic

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[*]Corresponding author

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equations is still an interesting work in the existing literature [6-9].

Recently, the study of exponential-type potentials has attracted much attention to many scientists both in non-relativistic quantum mechanics and in relativistic quantum mechanics [10-23]. These potentials include the Hulthen potential [10, 11, 20-23], the multi parameter exponential type potentials [12-15], the Manning-Rosen potential [22] and the Eckart-type potential [17, 18]. It should be mentioned that most contributions appearing in the literature concern with the s-wave case. However, for the l-wave, one can only solve approximately by using a suitable approximation scheme [24].

The Klein-Gordon equation with the vector and scalar potentials can be written as follows:

$$\left\lfloor \nabla^2 + [V(r) - E]^2 - [S(r) + M]^2 \right\rfloor \psi(r, \theta, \varphi) = 0 \tag{1}$$

where M is the rest mass, $i\frac{\partial}{\partial t} = E$ is the energy eigenvalues, $V(r)$ and $S(r)$ are the vector and scalar potential, respectively. Different methods such as the Supersymmetry Quantum Mechanics [25], Asymptotic Iteration Method (AIM) [26] and Nikiforov-Uvarov (NU) [27] and others have been used to solve the second order differential equations arising from these considerations.

The Hulthen potential [28] is one of the important molecule potentials with an exponential-type included a wide class of potentials in several branches of physical science. This potential is studied by many authors solving relativistic or non-relativistic differential type equations [29] and its deformed shape is defined by

$$U_v^0 = -Ze^2\delta \frac{e^{-\delta r}}{(1 - qe^{-\delta er})}, \tag{2}$$

where Z is a constant, δ and q are the screening and deformation parameters, respectively. For small r compared to 1δ, the Hulthen potential approaches the Coulomb potential whereas for large r compared to 1δ, it approaches to zero exponentially. Therefore, 1δ term can be thought as an infrared regulator for the Coulomb problem [30]. In addition, for $q = 1$ the deformed form reduces to the Hulthen potential form and for $q = -1$ to the standard Woods-Saxon potential.

Antia et al. [31] has obtained the bound state solution of the KGE under modified deformed Hulthen potential with position dependent mass for unequal scalar and vector potentials.

In this paper, the Deformed Generalized Hulthen potential will be solved for equal scalar and vector potentials using NU method.

The organization of this paper is as follows: In Section 2 and Section 3, we review the Klein-Gordon equation and Nikiforov-Uvarov method, respectively. In Section 4, we present the solutions of the Klein-Gordon equation for Deformed Generalized Hulthen potential and conclude in Section 5.

2. Review of the Klein-Gordon Equation

The Klein-Gordon equation is a relativistic wave equation, similar to the Schrödinger equation. It is second order in space and time and manifestly Lorentz covariant. It is a quantized version of the relativistic energy-momentum relation. Its solutions include a quantum scalar or pseudoscalar field, a field whose quanta are spinless particles. Its theoretical relevance is the same as that of the Dirac equation [32].

The Klein-Gordon equation with mixed scalar $S(r)$ and vector $V(r)$ potentials is given by [33]

$$\left\lfloor \nabla^2 + [V(r) - E]^2 - [S(r) + M]^2 \right\rfloor \psi(r, \theta, \varphi) = 0, \tag{3}$$

where M is the rest mass, E is the relativistic energy, ∇^2 is the Laplace operator. In the spherical coordinate, the Klein-Gordon equation for a particle is given by

$$\begin{bmatrix} \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial}{\partial\theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial\varphi^2} \\ - 2(EV(r) + MS(r)) + V^2(r) - S^2(r) + E^2 - M^2 \end{bmatrix} \psi(r, \theta, \varphi) = 0. \tag{4}$$

The total wave function can be written as

$$\psi(r, \theta, \varphi) = \frac{R(r)}{r} Y_{lm}(\theta, \varphi). \tag{5}$$

Eq. (4) is separated into variables and the following equations are obtained

$$\frac{d^2R(r)}{dr^2} + \left[E^2 - M^2 - 2(EV(r) + MS(r)) + V^2(r) - S^2(r) - \frac{\lambda}{r^2}\right]R(r), \quad (6)$$

$$\frac{d^2\Theta(\theta)}{d\theta^2} + \cot\theta \frac{d\Theta}{d\theta} + \left(\lambda - \frac{m^2}{\sin^2\theta}\right)\Theta(\theta) = 0, \quad (7)$$

$$\frac{d^2\Phi(\varphi)}{d\varphi^2} + m^2\Phi(\varphi) = 0, \quad (8)$$

where

$$Y_{lm}(\theta, \varphi) = \Theta(\theta)\Phi(\varphi). \quad (9)$$

m^2 and $\lambda = l(l+1)$ are the separation constants. Eqs. (7) and (8) are spherical harmonic functions $Y_{lm}(\theta, \varphi)$ whose solution are well known.

3. Review of the Nikiforov-Uvarov (NU) Method

Nikiforov and Uvarov [34] have presented the NU method to obtain the exact solution of the second order differential equations such as the Schrodinger, Klein-Gordon and Dirac equations.

The SE

$$\psi''(n) + (E - V(x))\psi(x) = 0 \quad (10)$$

can be solved by using this method. This can be done by transforming this equation of hypergeometric type with appropriate transformation, $s = s(x)$

$$\psi''(s) + \frac{\bar{\tau}(s)}{\sigma(s)}\psi'(s) + \frac{\bar{\sigma}(s)}{\sigma^2(s)}\psi(s) = 0. \quad (11)$$

In order to find the solution of Eq. (11), we set the wave functions as

$$\psi(s) = \phi(s)\chi(s) \quad (12)$$

and on substituting Eq. (12) into Eq. (11), Eq. (11) is reduced to hypergeometric type equation:

$$\sigma(s)\chi''(s) + \tau(s)\chi'(s) + \chi(s) = 0, \quad (13)$$

where the wave function $\phi(s)$ is defined as the logarithmic derivative.

$$\frac{\phi'(s)}{\phi(s)} = \frac{\pi(s)}{\sigma(s)}, \tag{14}$$

where $\pi(s)$ is at most first order polynomials, $\sigma(s)$ is at most a second order polynomials.

Likewise, the hypergeometric type function $\chi(s)$ in Eq. (13) for a fixed n is given by Rodrigues relation as

$$\chi_n(s) = \frac{B_n}{\rho(s)} \frac{d^n}{ds^n}[\sigma^n(s)\rho(s)], \tag{15}$$

where B_n is the normalization constant and the weight function $\rho(s)$ must satisfy the condition

$$\frac{d}{ds}(\sigma(s)\rho(s)) = \tau(s)\rho(s) \tag{16}$$

with

$$\tau(s) = \bar{\tau}(s) + 2\pi(s). \tag{17}$$

Therefore, the function $\pi(s)$ and the parameters required for the NU method are defined as follows:

$$\pi(s) = \frac{\sigma' - \bar{\tau}}{2} \pm \sqrt{\left(\frac{\sigma' - \bar{\tau}}{2}\right)^2 - \bar{\sigma} + k\sigma} \tag{18}$$

$$\lambda = k + \pi'(s). \tag{19}$$

The k-valuate if the expression under the square root be square of polynomials. This is possible, if and only if its discriminant is zero. With this, the new eigenvalues equation becomes

$$\lambda = \lambda_n = -\frac{nd\tau}{ds} - \frac{n(n-1)}{2}\frac{d^2\sigma}{ds^2}, \quad n = 0, 1, 2, \ldots . \tag{20}$$

On comparing Eq. (19) and Eq. (20), we obtain the energy eigenvalues.

The parametric generalization of NU method that is valid for any non-central potential is given by the generalized hypergeometric-type equation.

$$\psi''(s) + \frac{c_1 - c_2 s}{s(1 - c_3 s)} \psi'(s) + \frac{1}{s^2 (1 - c_3)^2} [-\xi_1 s^2 + \xi_2 s - \xi_3]\psi(s) = 0. \quad (21)$$

Equation (21) is solved by comparing it with Eq. (11) and the following polynomials are obtained

$$\bar{\tau}(s) = (c_1 - c_2 s), \quad \sigma(s) = s(1 - c_3 s), \quad \bar{\sigma}(s) = -\xi_1 s^2 + \xi_2 s - \xi_3. \quad (22)$$

Now substituting Eq. (22) into Eq. (18), we find

$$\pi(s) = c_4 + c_5 s \pm [(c_6 - c_3 k_\pm) s^2 + (c_7 + k_\pm) s + c_8]^{1/2}, \quad (23)$$

where

$$c_4 = \frac{1}{2}(1 - c_1), \quad C_5 = \frac{1}{2}(c_2 - 2C_3),$$

$$c_6 = c_5^2 + \xi_1, \quad c_7 = 2c_4 c_5 - \xi_2, \quad c_8 = c_4^2 + \xi_3. \quad (24)$$

The resulting value of k in Eq. (23) is obtained from the condition that the function under the square root is the square of a polynomials and it yields

$$k_\pm = -(c_7 + 2c_3 c_8) \pm 2\sqrt{c_8 c_9}, \quad (25)$$

where

$$c_9 = c_3 c_7 + c_2^2 c_8 + c_6, \quad (26)$$

the new $\pi(s)$ for each k becomes

$$\pi(s) = c_4 + c_5 s - \left\lfloor \left(\sqrt{c_9} + c_3\sqrt{c_8}\right) s - \sqrt{c_8} \right\rfloor \quad (27)$$

for the k_- value

$$k_- = -(c_7 + 2c_3 c_8) - a\sqrt{c_8 c_9}. \quad (28)$$

Using Eq. (17), we obtain

$$\tau(s) = c_1 + 2c_4 - (c_2 - 2c_5) s - 2\left\lfloor \left(\sqrt{c_9} + c_3\sqrt{c_8}\right) s - \sqrt{c_8} \right\rfloor. \quad (29)$$

The physical condition for the bound state solution $\tau' < 0$ and thus

$$\tau'(s) = -2c_3 - 2\left(\sqrt{c_9} + c_3\sqrt{c_8}\right) < 0. \tag{30}$$

Using Eqs. (19) and (20), we derive the energy equation as

$$(c_2 - c_3)n + c_3 n^2 - (2n+1)c_5$$
$$+ (2n+1)\left(\sqrt{c_9} + c_3\sqrt{c_8}\right) + c_7 + 2c_3c_8 + 2\sqrt{c_8c_9} = 0. \tag{31}$$

The weight function $\rho(s)$ is obtained from Eq. (16) as

$$\rho(s) = s^{c_{10}-1}(1 - c_3 s)^{\frac{c_{11}}{c_3} - c_{10} - 1} \tag{32}$$

and together with Eq. (15), we have

$$\chi_n(s) = P_n^{\left(c_{10}-1, \frac{c_{11}}{c_3} - c_{10} - 1\right)}(1 - 2c_3 s), \tag{33}$$

where

$$c_{10} = c_1 + 2c_4 + 2\sqrt{c_8}, \tag{34}$$

$$c_{11} = c_2 - 2c_5 + 2\left(\sqrt{c_9} + c_3\sqrt{c_8}\right) \tag{35}$$

and $P_n^{(\alpha,\beta)}(s)$ are the Jacobi polynomials. The second part of the wave function is obtained from Eq. (14) as

$$\phi(s) = s^{c_{12}}(1 - c_3 s)^{-c_{12} - \frac{c_{13}}{c_3}}, \tag{36}$$

where

$$c_{12} = c_4 + \sqrt{c_8}, \quad c_{13} = c_5 - \left(\sqrt{c_9} + c_3\sqrt{c_8}\right). \tag{37}$$

Thus, the total wave function becomes

$$\psi(s) = N_n s^{c_{12}}(1 - c_3 s)^{-c_{12} - \frac{c_{13}}{c_3}} P_n^{\left(c_{10}-1, \frac{c_{11}}{c_3} - c_{10} - 1\right)}(1 - 2c_3 s), \tag{38}$$

where N_n is the normalization constant.

4. Solutions of the Klein-Gordon Equation for Deformed Generalized Hulthen Potential

From Eq. (6), set $S(r) = V(r)$, we have

$$\frac{d^2R}{dr^2} + \left[E^2 - M^2 - 2(E+M)V(r) - \frac{\lambda}{r^2}\right]R(r) = 0. \tag{39}$$

The Deformed generalized Hulthen potential is defined as (31)

$$V(r) = \frac{V_0 e^{-2\propto r}}{1 - qe^{-2\propto r}} + \frac{V_1 e^{-2\propto r}}{(1 - qe^{-2\propto r})^2}. \tag{40}$$

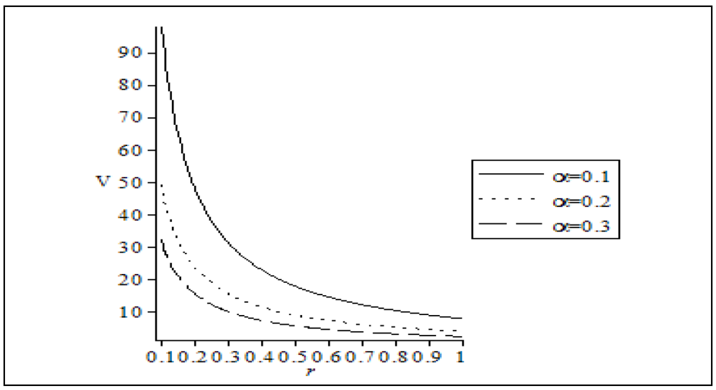

Figure 1. Variation of Deformed Generalized Hulthen potential with r for $V_0 = 2.0\,\text{MeV}$ and for values of $\alpha = 0.1,\ 0.2$ and 0.3.

Substituting Eq. (40) into Eq. (39), we have

$$\frac{d^2R}{dr^2} + \left[E^2 - M^2 - 2(E+M)\left(\frac{V_0 e^{-2\propto r}}{1 - qe^{-2\propto r}} + \frac{V_1 e^{-2\propto r}}{(1 - qe^{-2\propto r})^2}\right) - \frac{\lambda}{r^2}\right]R(r) = 0. \tag{41}$$

Eq. (41) has no analytical solution for $l \neq 0$ due to the centrifugal term. Therefore, we must take a proper approximation [35] to centrifugal term as

$$\frac{1}{r^2} \approx 4\alpha^2\left(c_0 + \frac{e^{-2\alpha r}}{(1 - qe^{-2\alpha r})^2}\right), \tag{42}$$

where $C_0 = \frac{1}{12}$ is an arbitrary dimensionless constant. In this study, we set $C_0 = 0$

which reduces Eq. (42) to convectional approximation scheme [36]. The choice of $C_0 = 0,$ does not alter the physics of the problem under investigation [31].

Substituting Eq. (42) into (41), $C_0 = 0,$ we have

$$\frac{d^2R}{dr^2} + \left[E^2 - M^2 - \frac{2(E+M)V_0 e^{2\alpha r}}{1 - qe^{-2\alpha r}} - \frac{2(E+M)V_1 e^{-2\alpha r}}{(1 - qe^{-2\alpha r})^2} - \frac{4\alpha^2 e^{-2\alpha r}}{(1 - qe^{-2\alpha r})^2}\right] R(r) = 0. \tag{43}$$

Let $s = e^{-2\alpha r}.$ (44)

Substituting Eq. (44) into Eq. (43), we have

$$\frac{d^2R}{ds^2} + \frac{(1-qs)}{s(1-qs)}\frac{dR}{ds} + \frac{1}{4\alpha^2 s^2 (1-qs)^2}\begin{bmatrix} s^2(E^2q^2 - M^2q^2 + 2(E+M)V_0 q) + s \\ \begin{pmatrix} -2E^2q + 2M^2q - 2(E+M)V_0 \\ -2(E+M)V_1 - 4\alpha^2\lambda \end{pmatrix} \\ + E^2 - M^2 \end{bmatrix} R(s) = 0. \tag{45}$$

$$\text{Set } -\beta^2 = \frac{E^2 - m^2}{4\alpha^2};\ X = \frac{-2(E+M)V_0 q}{4\alpha^2};\ W = \frac{1}{4\alpha^2}[-2(E+M)V_0 - 2$$

$$\times (E+M)V_1 - 4\alpha^2\lambda]. \tag{46}$$

Substituting Eq. (46) into Eq. (45), we have

$$\frac{d^2R}{ds^2} + \frac{(l - qs)}{s(l - qs)}\frac{dR}{ds} + \frac{1}{4\alpha^2 s^2 (l - qs)^2}[-(q^2\beta^2 + X)s^2$$

$$+ (2q\beta^2 + W)s - \beta^2]R(s) = 0. \tag{47}$$

Comparing Eq. (47) to Eq. (21)

$$\xi_1 = q^2\beta^2 + X;\ \xi_2 = 2q\beta^2 + W;\ \xi_3 = \beta^2, \tag{48}$$

$c_1 = 1,\ c_2 = q,\ c_3 = q,\ c_4 = 0,\ c_5 = -q/2,\ c_6 = q^2/4 + \xi_1,\ c_7 = -\xi_2,\ c_8 =$

$$\xi_3,\ c_9 = \xi_1 - q\xi_2 + q^2\xi_3 + {q^2}/{4}\ . \tag{49}$$

Substituting Eq. (49) into Eq. (31), we have

$$qn^2 + (2n+1)\frac{q}{2} + (2n+1)\left[\sqrt{\xi_1 - q\xi_2 + q^2\xi_3 + \frac{q^2}{4}} + q\sqrt{\xi_3}\right]$$

$$-\xi_2 + 2q\xi_3 + 2\sqrt{\xi_3\left(\xi_1 - q\xi_2 + q^2\xi_3 + \frac{q^2}{4}\right)} = 0, \tag{50}$$

$$\xi_1 - q\xi_2 + q^2\xi_3 + \frac{q^2}{4} = X - qW + \frac{q^2}{4} = \frac{2(E+M)}{4\alpha^2} + \lambda q + \frac{q^2}{4} = A^2, \tag{51}$$

$$-\xi_2 + 2q\xi_3 = -\frac{2(E+M)(V_0+V_1)}{4\alpha^2} + \lambda = B. \tag{52}$$

Let

$$P = -qn^2 - \frac{(2n+1)}{2}q. \tag{53}$$

Substituting Eqs. (51-53) into Eq. (50), we have

$$(2n+1)(A+q\beta) + \beta + 2\beta A = P. \tag{54}$$

Rearranging Eq. (54) and squaring both sides, we have

$$E^2 - M^2 = -\frac{4\alpha^2[(P-B)^2 - 2(P-B)(2n+1)A + (2n+1)^2A^2]}{[(2n+1)^2q^2 + 4(2n+1)qA + 4A^2]}. \tag{55}$$

The weight function $\rho(s)$ is obtained from Eq. (32)

$$\rho(s) = S^{2\beta}(1-qs)^{1+\frac{2}{q}\sqrt{\xi_1 - q\xi_2 + q^2\xi_3 + \frac{q^2}{4}}}, \tag{56}$$

where

$$c_{10} = 1 + 2\beta, \tag{57}$$

$$c_{11} = 2q + 2\left(\sqrt{\xi_1 - q\xi_2 + q^2\xi_3 + \frac{q^2}{4}} + q\beta\right), \tag{58}$$

$$c_{12} = \sqrt{\xi_3} = \beta, \tag{59}$$

$$c_{13} = -\frac{q}{2} - \left(\sqrt{\xi_1 - q\xi_2 + q^2\xi_3 + \frac{q^2}{4}} + q\beta\right). \tag{60}$$

Also using Eq. (33) we have

$$\chi_n(s) = P_n^{\left(2\beta, 1+\frac{2}{q}\sqrt{\xi_1 - q\xi_2 + q^2\xi_3 + \frac{q^2}{4}}\right)}(1-2qs). \tag{61}$$

The other ware function $\phi(s)$ is obtained using Eq. (36)

$$\phi(s) = S^{\beta}(1-qs)^{\frac{1}{2}+\frac{1}{q}\sqrt{\xi_1 - q\xi_2 + q^2\xi_3 + \frac{q^2}{4}}}. \tag{62}$$

The total wave function is obtained from Eq. (38)

$$\psi(s) = N_n S^{\beta}(1-qs)^{\frac{1}{2}+\frac{1}{q}\sqrt{\xi_1 - q\xi_2 + q^2\xi_3 + \frac{q^2}{4}}} P_n^{\left(2\beta, 1+\frac{2}{q}\sqrt{\xi_1 - q\xi_2 + q^2\xi_3 + \frac{q^2}{4}}\right)}(1-2qs). \quad 63)$$

Setting $M_1 = 0$ in Antia et al. [31], and $V_1 = 0$ and $S_0 \neq V_0$ in our work, we find out that our work is in agreement with theirs. The numerical values of the energy eigenvalues for this system is presented in Table 1. From the table it can be seen that the values confirm the bound state nature of the system as they are all negative which is a necessary condition for the bound state energy for particles. The behaviour of the potential model is discussed in Figure 1.

Table 1. Bound State Energy of Deformed Generalized Hulthen Potential for $M = 1, V_0 = 0.02, V_1 = 0.03$

$\lvert n, l\rangle$	Energy levels for $q = 0.1$			Energy levels for $q = 0.5$		
	$\alpha = 0.02$	$\alpha = 0.04$	$\alpha = 0.06$	$\alpha = 0.02$	$\alpha = 0.04$	$\alpha = 0.06$
1,0	-0.9996771930	-0.9987084251	-0.9970926538	-0.9998673305	-0.9994692398	-0.9988054803
1,1	-0.9997396766	-0.9989584372	-0.9976554722	-0.9999394115	-0.9997576249	-0.9994545764
1,2	-0.9998215218	-0.9992859319	-0.9983927640	-0.9999682866	-0.9998731403	-0.9997145432
1,3	-0.9998836307	-0.9995344487	-0.9989522312	-0.9999806758	-0.9999227010	-0.9998260690
2,0	-0.9997194152	-0.9988773827	-0.9974730661	-0.9999259748	-0.9997038704	-0.9993336005

2,1	-0.9997758863	-0.9991033340	-0.9979817078	-0.9999623032	-0.9998492043	-0.9996606783
2,2	-0.9998464486	-0.9993856755	-0.9986173228	-0.9999778938	-0.9999115722	-0.9998010265
2,3	-0.9998983960	-0.9995935264	-0.9990852188	-0.9999855154	-0.9999420602	-0.9998696307
3,0	-0.9997557027	-0.9990225882	-0.9977999867	-0.9999544562	-0.9998178134	-0.9995900369
3,1	-0.9998064743	-0.9992257313	-0.9982572734	-0.9999744196	-0.9998976744	-0.9997697528
3,2	-0.9998670872	-0.9994682570	-0.9988032335	-0.9999837273	-0.9999349077	-0.9998535365
3,3	-0.9999106749	-0.9996426547	-0.9991958044	-0.9999887430	-0.9999549712	-0.9998986824
4,0	-0.9997868149	-0.9991470814	-0.9980802638	-0.9999696464	-0.9998785804	-0.9997267861
4,1	-0.9998322666	-0.9993289365	-0.9984896191	-0.9999815439	-0.9999261737	-0.9998338833
4,2	-0.9998842313	-0.9995368539	-0.9989576536	-0.9999875277	-0.9999501098	-0.9998877435
4,3	-0.9999209602	-0.9996838052	-0.9992884283	-0.9999910017	-0.9999640063	-0.9999190123

5. Conclusion

The exact and approximate solutions of Klein-Gordon equation with Deformed Generalized Hulthen potential for equal scalar and vector fields using generalized parametric NU method has been considered by using an improved approximation scheme (Jia et al. [35]). The energy eigenvalues and the corresponding wave functions expressed in terms of the Jacobi polynomials have been obtained. The numerical values of our results have also been computed. The results obtained in this paper agree with those available in the existing literature.

References

[1] L. I. Schiff, Quantum Mechanics, McGraw Hill, New York, 1955.

[2] P. A. Dirac, The principles of Quantum Mechanics, Oxford University Press, USA, 1958.

[3] L. D. Landau and E M Lifshitz, Quantum Mechanics, Non-Relativistic Theory, Pergamon, Canada, 1977.

[4] W. Greiner, Quantum Mechanics - An Introduction, Springer, Berlin, 1989.

[5] S. H. Dong, Phys. Scr. 67 (2003), 89.

[6] A. N. Ikot, L. E. Akpabio and J. A. Obu, J. Vect. Relat 6 (2011), 1.

[7] W. C. Qiang and S. H. Dong, Phys. Lett. A 372 (2008), 4789.

[8] X. C. Zhang, Q. W. Liu, C. S. Jia and L. Z. Wang, Phys. Lett. A 340 (2005), 59.

[9] W. C. Qiang and S. H. Dong, Phys. Scr. 79 (2009), 045004.

[10] O. Bayrak, G. Kocak and I. Boztosun, J. Phys. A Math. Gen. 39 (2006), 11521.

[11] B. Gonul, O. Ozer, Y. Cancelik and M. Kocak, Phys. Lett. A275 (2000), 238.

[12] H. Egrifes, D. Demirhan and F. Buyukkilic, Phys. Lett. A 275 (2000), 229.

[13] C. S. Jia, Y. F. Diao, L. Min, Q. B. Yang, L. T. Sun and R. Y. Huang, J. Phys. A Math. Gen. 37 (2004), 11275.

[14] C. S. Jia, Y. Li., Y. Sun, J. Y. Liu and L. T. Sun, Phys. Lett. A 311 (2003), 115.

[15] C. S Jia, X. L. Zeng and L. T. Sun, Phys. Lett. A 294 (2002), 185.

[16] S. H. Dong and J. Garcia-Ravelo, Phys. Scr. 75 (2007), 307.

[17] X. Zou, L. Z. Yi and C. S. Jia, Phys. Lett. A 346 (2005), 54.

[18] C. S. Jia, P. Guo and X. L. Peng, J. Phys. A Math. Gen. 39 (2006), 7737.

[19] R. L. Greene and C. Aldrich, Phys. Rev. A 14 (1976), 2363.

[20] F. Dominguez-Adame, Phys. Lett. A 136 (1989), 175.

[21] G. Chen, Z. D. Chen and Z. M. Lou, Phys. Lett. A 331 (2004), 374.

[22] S. Mehmet and E. Harun, J. Phys. A Math. Gen. 37 (2004), 4379.

[23] L. Chetouani, L. Guechi, A. Lecheheb, T. F. Hammann and A. Messouber, Physica A 234 (1996), 529.

[24] J. Lu, Phys. Scr. 72 (2005), 349.

[25] C. S. Jia, P. Gao and X. L. Peng, Exact solution of the Dirac-Eckart problem with spin and pseudospin symmetry, J. Phys. A Math. Gen. 39 (2006), 7737.

[26] H. Ciftci, R. L. Hall and N. Saad, Asymptotic iteration method for eigenvalue problems, J. Phys. A 36 (2003), 1807.

[27] Y. F. Cheng and T. O. Dai, Exact solution of the Klien-Gordon equation with a ring - shape modified Kratzer potential, Chin. J. Phys. 45 (2007), 480-487.

[28] L. Hulthen, Ark. Mat. Astron. Fys. 28A (1942), 5.

[29] N. A. Rao and B. A. Kagali, Physics Letters A 296 (2002), 192.

[30] M. Znojil, Physics Letters A 102 (1984), 289.

[31] Akaninyene D. Antia, Akpan N. Ikot, Eno E. Ituen and Ita O. Akpan, Bound state solutions of the Klein-Gordon equation for deformed Hulthen potential with position dependent mass, Sri Lankan J. Phys. 13(1) (2012), 27-40.

[32] Y. Xu, S. He and C. S. Jia, Approximate analytical solutions of the Klein-Gordon equation with the Poschl-Teller potential including the centrifugal term, Phys. Scr. 81 (2010), 045001.

[33] A. S. Davydov, Quantum Mechanics, 2nd ed., Pergamon Press, 1976.

[34] A. F. Nikiforov and V. B. Uvarov, Special Functions of mathematical Physics, Birkhauser, Basel, 1988.

[35] S. Jia, C. T. Chen and G. Cui, Phys. Lett. A. 373 (2009), 1621.

[36] R. L. Greene and C. Aldrich, Phys. Rev. A. 14 (1976), 2363.