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SCALE-FREE PROPERTY OF PERSISTENT LENGTH DISTRIBUTION IN ANOMALOUS DIFFUSIONS

HYUN-JOO KIM

Department of Physics Education
Korea National University of Education
Chungbuk 363-791
Korea
e-mail: hjkim21@knue.ac.kr

Abstract

To describe the property of the anomalous diffusion, we consider the distribution of persistent length which is the number of successive steps in the same direction. We used the non-Markovian models showing the anomalous diffusions by enhancing the memory for the previous step with the time and found that the persistent length distribution follows the power-law behavior $p(s) \sim s^{-\beta}$ with the exponent $\beta \approx 2$ within a characteristic length which depends exponentially on the Hurst exponent. It indicates that the scale-free property of the persistent length distribution might be a key describing the underlying mechanism of the anomalous diffusions.

1. Introduction

Anomalous diffusion phenomena have attracted considerable attentions in the field of statistical physics for few decades [1-4]. It is compared to the well-known

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normal diffusion which is described by the random walk problem [5]. The key separating the normal and the abnormal is the different power-law behavior of the fluctuation of displacement, i.e., the quantity, the mean squared displacement (MSD). The MSD grows linearly with time for the normal diffusion, however, it behaves in nonlinear way for the anomalous diffusions which have been observed in many different systems such as hydrologic [6-7], chaotic [8], biophysical [9-12], and economic systems [13-14], etc. The nonlinear behavior of the MSD $\langle x^2(t) \rangle$ is characterized as

$$\langle x^2(t) \rangle \sim t^{2H}. \tag{1}$$

Here $\langle \cdots \rangle$ means average over independent realizations, i.e., ensemble average, in general, in non-equilibrium. H is called as the anomalous diffusion or the Hurst exponent. It classifies superdiffusion in which $H > 1/2$ and thus the past and future random variables are positively correlated and thus persistence is exhibited, and subdiffusion in which $0 < H < 1/2$ and the random variables are negatively correlated showing antipersistence.

The representative models describing the underlying mechanism of anomalous diffusions are the fractional Brownian motion (fBM) [1], the Lévy flights [4], and the continuous time random walks (CTRW) [2, 3]. Recently, various microscopic non-Markovian models with memory effect which may be a key origin were proposed [15-18]. Above mentioned various models propose the different origins for anomalous diffusions separately, however they do not give any universal mechanism of the nonlinearity of the MSD. Therefore, it is necessary to consider another aspect to be able to characterize the anomalous diffusive phenomena. The nonlinearity of the MSD is related to the persistent (or antipersistent) behavior of a walker so that it is meaningful to investigate the property of the persistent behavior more directly. Therefore, in this work, we study the characteristics of persistent (or antipersistent) behavior in anomalous diffusion by measuring the distribution of the persistent length using the models with time-varying correlations between the past and the future steps which describe well the nature of superdiffusion and subdiffusion both [18].

2. Models

The non-Markovian stochastic model in which the memory of the previous step is enhanced with time was used [18]. The rule of the model is given as follows. The random walker starts at origin and randomly moves either one step to the right or the left at time $t = 1$, so the position of the walker becomes $x_1 = \sigma_1$, where σ is random variable with the value of $+1$ or -1. Then for $t > 1$, σ_{t+1} is given by

$$\sigma_{t+1} = \begin{cases} \sigma_t, \text{ with probability } 1 - 1/t^{\alpha}, \\ 1 \text{ or } -1, \text{ with probability } 1/t^{\alpha}. \end{cases} \tag{2}$$

Over time, the probability of taking the same direction with the latest step increases and the larger value of parameter α is, the much faster the probability grows with time. That is, in this model the persistence with the previous step is enhanced with time of which degree is controlled by the parameter α. When $\alpha = 0$, it is reduced to the original random walk. We shall refer to this model as the model A. The relation between the Hurst exponent H and the parameter α is given by $2H = 1 + \alpha$ showing superdiffusive behaviors in the model A [18]. Meanwhile in Eq. (2) if the rule $\sigma_{t+1} = -\sigma_t$ is taken, the correlation between two successive steps is negative and thus antipersistence is enhanced with time. We call it the model B in which subdiffusion is shown and the relation $2H = 1 - \alpha$ is given.

3. Results

We measured the distribution $p(s)$ of persistent lengths which is the number of the successive steps in the same direction. Figure 1 shows the plot of the cumulative distribution $p(s)$ of the psersistent length versus the persistent length s for the model A with the various control parameter α. The cumulative distribution is defined as $P(s) = \int_s^{\infty} dsp(s)$. The solid line is the guide line whose slope is 1, which indicates that the cumulative distribution follows the power-law,

$$P(s) \sim s^{-\beta+1} \tag{3}$$

with $\beta \approx 2$ and thus the distribution $p(s)$ scales as $p(s) \sim s^{-\beta}$. However, the range of s following the power-law is dependent on the control parameter α. The larger α is, the longer the range is. It indicates that the nonlinearity of the MSD results from the power law behavior of the persistent length distribution and the Hurst exponent is related to the range following the power law. The quantitative relation between the range and the Hurst exponent can be found out from the scaling function of the persistent length distribution.

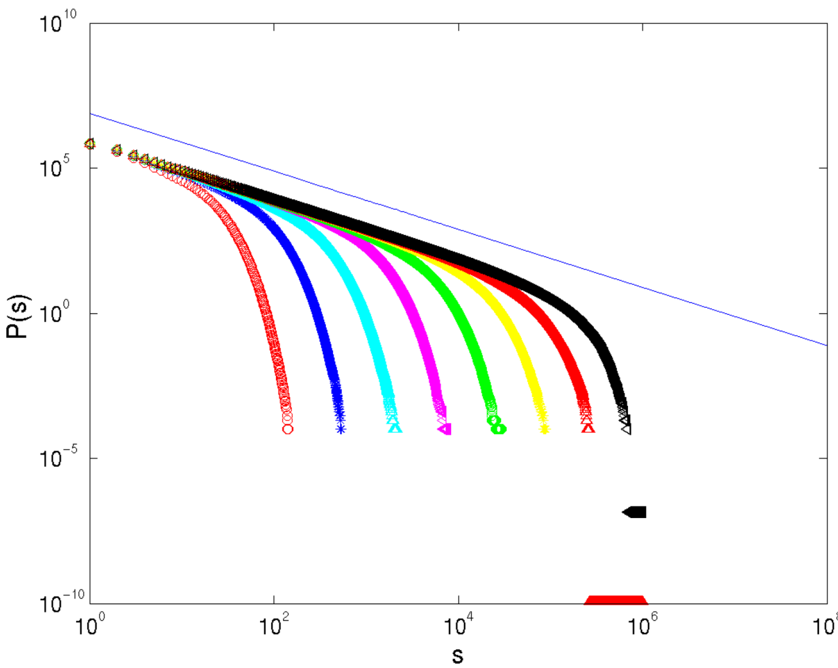

Figure 1. The cumulative distribution of persistent length $P(s)$ vs s for the model A with the control parameter $\alpha = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7,$ and 0.8 from the left to the right.

We found that the cumulative distribution of the persistent length $P(s, \alpha)$ for the various α obeys a scaling form of the following type,

$$P(s, \alpha) = s^{-\beta+1} f\left(\frac{s}{s_c(\alpha)}\right). \tag{4}$$

$f(x)$ is a scaling function satisfying the followings:

$$f(x) = \text{const.}, \quad \text{for } s \ll s_c(\alpha),$$

$$f(x) \to 0, \quad \text{for } s \gg s_c(\alpha). \tag{5}$$

$s_c(\alpha)$ is a characteristic length. Figure 2 shows the data collapse of the distribution of the persistent length $P(s, \alpha)$ with $\beta = 2$ and the characteristic length $s_c(\alpha)$ which is given by

$$s_c(\alpha) \sim \exp(A\alpha), \tag{6}$$

where A is a constant with the value $A = 13.1(2)$. They fall on a single curve very well for various α. It indicates that the characteristic length s_c grows exponentially with the parameter α and from the relation between α and H, s_c behaves as with the Hurst exponent H in the model,

$$s_c(\alpha) \sim \exp[A(2H - 1)]. \tag{7}$$

Thus the exponentially growing power-law range strengthens the superdiffusive behavior. For the model B, it is meaningful to measure the antipersistent lengths which are the number of successive steps changing the direction at each step. Figure 3 shows the plot of the cumulative distribution $P(s)$ of antipersistent lengths for the model B with the various parameters α. The solid line is the guide line whose slope is 1. It indicates that the distribution follows the power-law behavior like the persistent length distribution, which results from the same method in enhancing the memory with time although the behavior of a walker is oppositely directed to the previous step. It also shows the same scaling behavior of the cumulative distribution of antipersistent lengths like Eq. (4). Figure 4 shows the data collapse of the cumulative distribution of antipersistent lengths $P(s, \alpha)$ with $\beta = 1$ and $A = 15.7(2)$. They also fall on a single curve very well for various α.

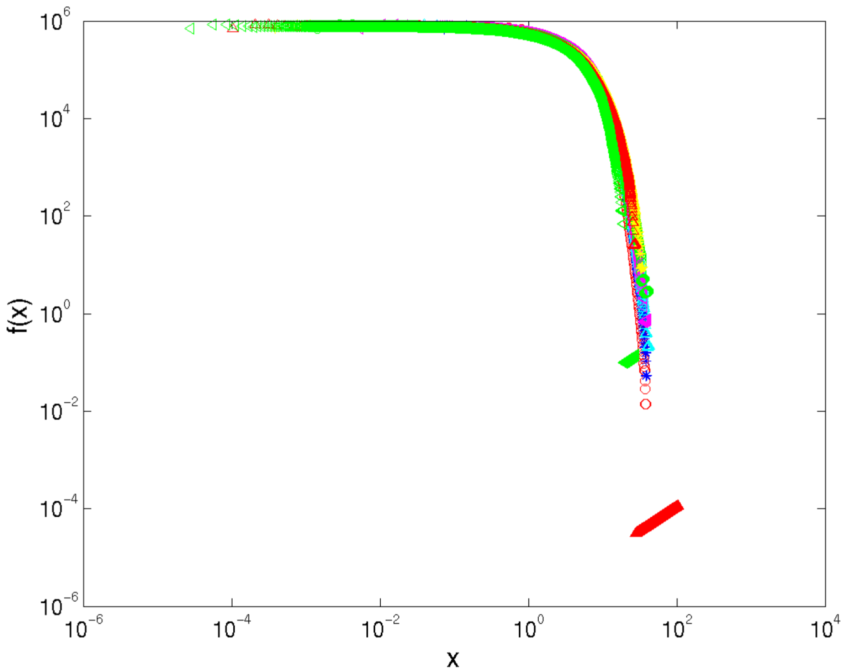

Figure 2. The data collapse of the cummulative distribution of the persistent length.

5. Conclusion

In conclusion, we have measured the distribution of persistent (antipersistent) length in superdiffusion (subdiffusion) by using the non-Markovian model with the memory enhancement with time for the previous step. It is found that the persistent (antipersistent) length distribution follows the power-law behavior with the exponent 2 within the characteristic range which depend on the Hurst exponent exponentially. Although it may be a result due to the specific property of the rule of the model in which the memory for the previous step is enhanced with the power of the time, it proposes the possibility of that the nonlinearity of the MSD in anomalous diffusions is induced by the power-law behavior of the persistent (antipersistent) length. And thus it is necessary to study further the generalization of the effect of the persistent (antipersistent) length on anomalous diffusions.

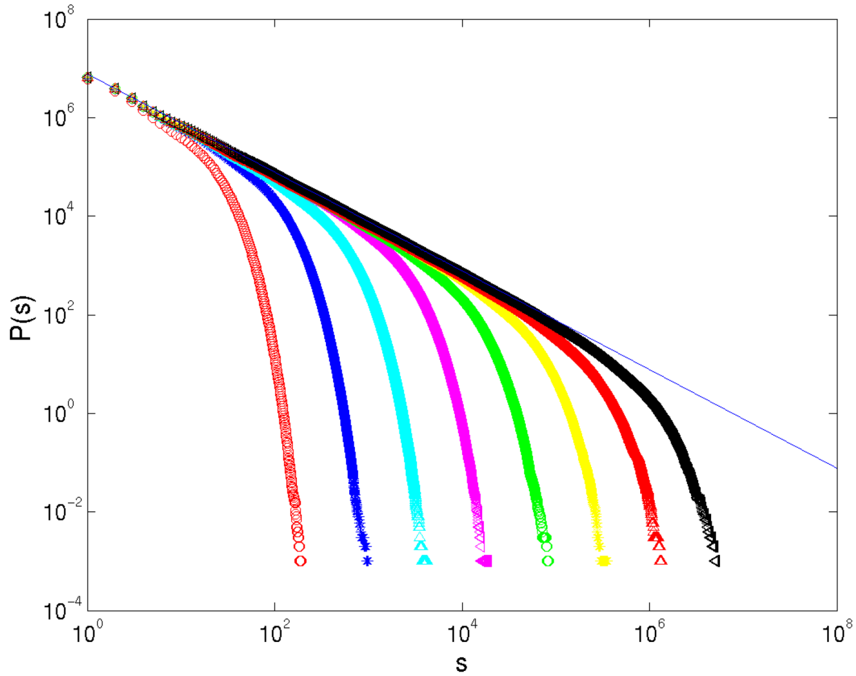

Figure 3. The cumulative distribution of antipersistent length $P(s)$ vs s for the model A with the control parameter $\alpha = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7,$ and 0.8 from the left to the right.

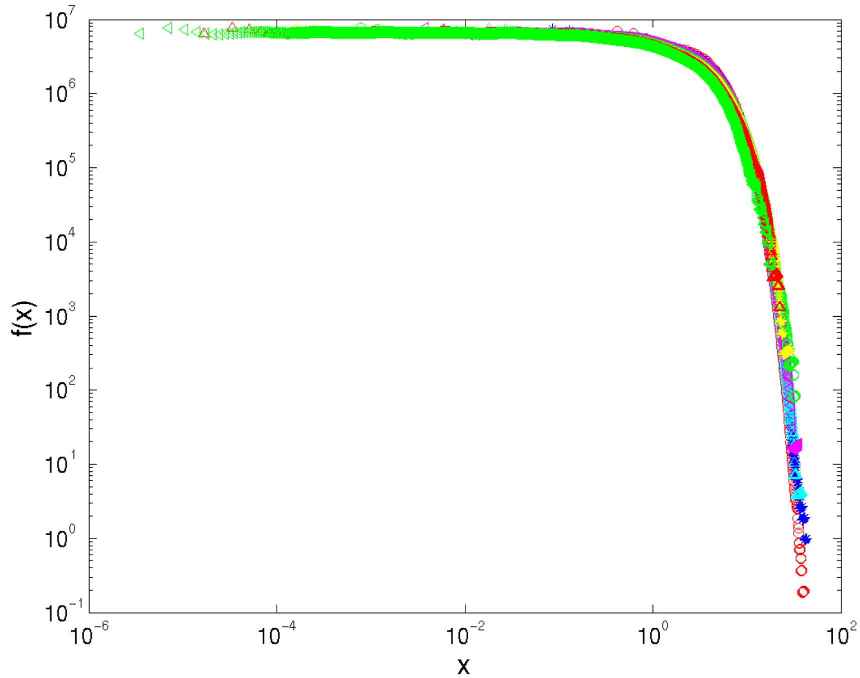

Figure 4. The data collapse of the cummulative distribution of the antipersistent length.

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