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RETARDED FORCES VIOLATE NEWTON'S FIRST AND THIRD LAWS OF MOTION

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Abstract

An analysis of non-relativistic collinear motion of two massive objects constrained by attractive inverse-square-law forces shows that, only in the case of an instantaneous interaction, as in Newton's theory of gravity or the Coulombic interaction between electrical charges, are Newton's First and Third mechanical laws respected. The breakdown of energy and momentum conservation, predicted by forces transmitted at the speed of light, is quantitatively estimated for two specific systems, one with gravitational the other with electromagnetic forces. Indirect experimental evidence for the non-retarded nature of gravitational forces, as well as recent direct experimental evidence for non-retarded magnetic and electric force fields, is recalled.

The Coulomb force in electrostatics and the inverse-square-law gravitational force in Newton's theory are transmitted instantaneously. Such behaviour is understood if the forces are due to the mediation of space-like virtual particles -

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photons or gravitons. The space-like character (negative mass squared) of the particles is essential for this explanation. A theory of this type, for electromagnetic forces, has been developed by the present author as the classical limit of quantum electrodynamics, the underlying fundamental process being Møller scattering: $ee \to ee$ [1]. Indeed, it is found that in the centre-of-mass system of the scattered electrons the interaction is an instantaneous one. On the other hand, the speed of all real (on-mass-shell) particles is restricted by relativistic kinematics to be, in an appropriate preferred frame, less than (or equal to, for massless particles) the speed of light. This is the case for photons produced by internal bremsstrahlung corrections to incoming or outgoing lines of Feynman diagrams, or in radiative transitions of atoms or molecules.

Laplace set a lower limit on the speed of gravity within a 'pushing gravity' model in which gravitational forces are attributed to the pressure of a spherically symmetric flux of radiation directed inwards towards the centre of the gravitational source. Due to aberration, a non-conservative drag force acts on any object in planetary motion in the gravitational field slowing its orbital motion. By considering the stability of the Moon's orbit around the Earth, Laplace concluded that the effective speed of the equivalent retarded force must be a factor of at least 10^8 times greater than the speed of light in free space [2].

As pointed out by Eddington [3], retarded spherically symmetric gravitational forces in a planetary system (he considered the Sun-Jupiter system) produce an unbalanced torque that violates Newton's third law of motion. The effect is a geometrical one that arises when two objects A and B are rotating about their common center-of-mass system. Motion of object B gives a non radial (with respect to the common centre of rotation) force on object A, and *vice versa*. These forces give an accelerating torque following from violation of Newton's third law of motion that leads to breakdown of angular momentum conservation. Drawing an analogy with the Heaviside 'present time' formula [4] for the electric field of a uniformly moving charge, which is radial, Eddington stated that there is 'very approximately' no breakdown of angular momentum conservation, insofar as planetary trajectories are approximately linear over a time interval equal to the 'time of flight' of a retarded force. However, angular momentum conservation for a classical planetary system is a law of physics that is either respected or not, it cannot be 'very approximately' respected. Applying this effect to the stability of binary pulsar systems, Van Flandern [5] found a lower limit for the speed of gravitational forces, due to the absence of any modification of the pulsar periods due to a putative unbalanced torque, of 2×10^{10} times the speed of light [5].

A calculation by the present author [6] demonstrated the impossibility of stable circular Keplerian orbits under electromagnetic forces derived from retarded Lienard-Wiechert fields [7, 8]. Recent experiments on magnetic [9, 10] and electric [11, 12] force fields have shown propagation speeds much greater than the speed of light, consistent with a non-retarded interaction.

The work presented here considers the simple case of collinear motion under the action of retarded, attractive, inverse-square-law forces. In this case the breakdown of Newton's First and Third Laws is particularly evident. Both gravitational and electromagnetic forces are considered in the applications of the calculations: For gravity, collinear motion of objects with the masses of the Earth and the Moon; for electromagnetic forces a proton and an electron. To simplify the numerical calculations it is assumed, in both cases, that the objects are initially receding from each other with the escape velocity appropriate for instantaneous forces. The calculations are performed in the context of non-relativistic classical mechanics where conservation of mass, kinetic energy and momentum are expected, that of the latter two due Noether's Theorem. Violation of energy and momentum conservation is, however, permitted in certain general-relativity-inspired cosmological models such as the FLRW Universe.

Consider two massive objects 1 and 2 subjected to mutual attractive inverse square law forces. They move in opposite directions along a straight line (x-coordinate axis) with a fixed direction in space. The objects, of mass m_1 and m_2, have velocities $v_1(t)$ and $v_2(t)$ such that the x-coordinate, x_B, of the barycenter of the composite body constituted by the two moving objects is at rest:

$$m_1 v_1(t) = m_2 v_2(t) = p(t). \tag{1}$$

It is assumed in this equation and the following calculations that $v_1, v_2 << c,$ where

c is the speed of transmission of the retarded forces and only terms of first order in $\beta \equiv v/c$ are retained. Newton's First Law applied to the composite body requires that, since no external forces act, x_B is constant. A space-time plot showing the world-lines of 1 and 2 at epoch t when they are distant $r_1(t)$, $r_2(t)$ from the barycenter is shown in Figure 1 with:

$$\frac{r_1(t)}{r_2(t)} = \frac{v_1(t)}{v_2(t)} = \frac{m_2}{m_1} = 2. \tag{2}$$

The effect the retarded forces exert on the motion of the objects is now considered. The retardation effects are calculated as an order β (i.e., small) correction to motion under the instantaneous forces of Newtonian mechanics. These forces are given by the $c \to \infty$ limit of the retarded force law.

In Figure 1, the world lines of the retarded force fields acting on 1 and 2, respectively, are denoted by γ_1 and γ_2. These retarded forces are equal to $F_1 = K/R_1^2$ and $F_2 = K/R_2^2$, where K is the constant of the force law. Since $R_1 > R_2$, the force acting on 2 at the epoch t is greater than that acting on 1 at the same epoch. Action and reaction are therefore unequal, violating Newton's Third Law. Also since a net force acts on the composite body, in the direction of the velocity of the lighter object, then if, Newton's Second Law holds, it will be accelerated in this direction. However, no external force acts, so Newton's First Law is violated.

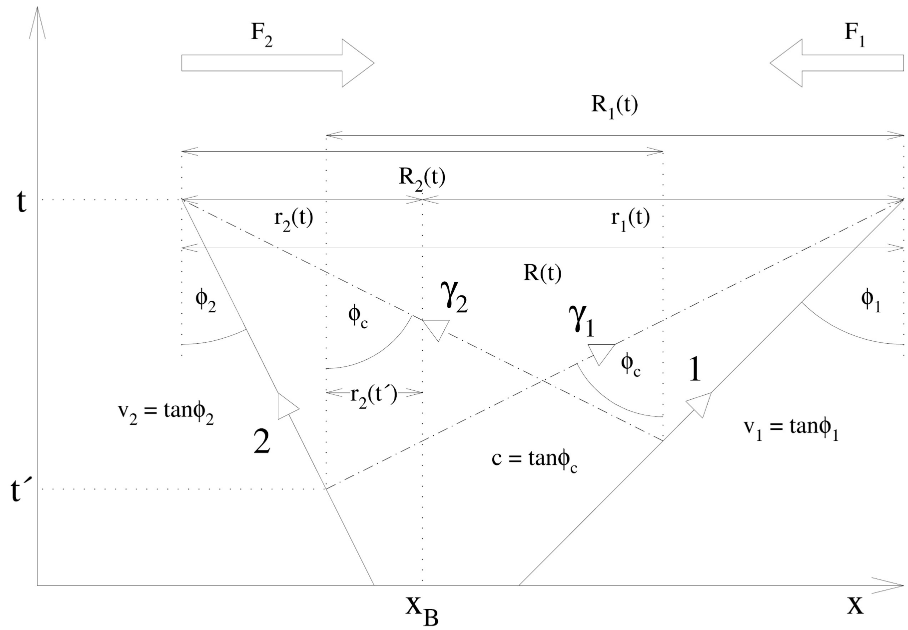

Figure 1. Space-time plot for the motion of two objects, 1 and 2, of mass m_1 and m_2 $(m_2 = 2m_1)$ under the influence of retarded forces F_1 and F_2 at epoch *t*. The momenta of the objects at this epoch are equal and opposite. γ_1 and γ_2 show the world lines of signals moving parallel to the directions of the retarded force vectors at the same speed *c*. See text for discussion of the various geometrical and kinematical quantities indicated.

Denoting the epoch of generation of F_1 as t', it can be seen from the geometry of Figure 1 that:

$$c(t - t') = r_1(t) + r_2(t'), \tag{3}$$

$$v_2(t - t') = r_2(t) - r_2(t'). \tag{4}$$

Combining (3) and (4), rearranging and making use of the condition (2) gives:

$$r_2(t') = \frac{r_1(t)}{m_2}\left[\frac{m_1 - m_2\beta_2}{1 + \beta_2}\right] \tag{5}$$

so that

$$R_1 \equiv r_1(t) + r_2(t') = \frac{r_1(t)}{m_2}\left[\frac{m_1 + m_2}{1+\beta_2}\right]. \tag{6}$$

Giving, for the ratio of the forces:

$$\frac{F_1}{F_2} = \frac{R_2^2}{R_1^2} = \left\{\frac{r_2(t)m_2}{r_1(t)m_1}\left[\frac{1+\beta_2}{1+\beta_1}\right]\right\}^2 = \left[\frac{1+\beta_2}{1+\beta_1}\right]^2. \tag{7}$$

According to Newton's Second Law the equations of motion of the objects are:

$$\frac{dp_1}{dt} \equiv \frac{d(m_1 v_1)}{dt} = \frac{-K}{R_1^2} = \frac{-K}{r_1(t)^2}\left[\frac{m_2(1+\beta_2)}{m_1 + m_2}\right]^2, \tag{8}$$

$$\frac{dp_2}{dt} \equiv \frac{d(m_2 v_2)}{dt} = \frac{-K}{R_2^2} = \frac{-K}{r_2(t)^2}\left[\frac{m_1(1+\beta_1)}{m_1 + m_2}\right]^2. \tag{9}$$

Introducing the equivalent one-body description of the two-body system [13] by making use of the relations, following from (2): $r_1(t) = mR/m_1$, $r_2(t) = mR/m_2$, where (see Figure 1) $R \equiv r_1(t) + r_2(t)$ and m is the reduced mass: $m \equiv m_1 m_2/(m_1 + m_2)$ enables to write (8) and (9) (neglecting β_1^2 and β_2^2 terms) as:

$$\frac{dp_1}{dt} = \frac{-K(1+\beta_2)^2}{R^2} \simeq \frac{-K}{R^2} - \frac{2K\beta_2}{R^2} \equiv F_1^{\text{inst}} + F_1^{\text{ret}}, \tag{10}$$

$$\frac{dp_2}{dt} = \frac{-K(1+\beta_1)^2}{R^2} \simeq \frac{-K}{R^2} - \frac{2K\beta_1}{R^2} \equiv F_2^{\text{inst}} + F_2^{\text{ret}}. \tag{11}$$

$F_1^{\text{inst}} = F_2^{\text{inst}} = F^{\text{inst}} = -K/R^2$ are the balanced instantaneous components of the attractive forces acting on the objects while

$$F_1^{\text{ret}} = \frac{-2K\beta_2}{R^2}, \quad F_2^{\text{ret}} = \frac{-2K\beta_1}{R^2}$$

are the unbalanced retarded components.

Integration of (10) gives:

$$\Delta p_1 = \int F_1^{\text{inst}} dt + \int F_1^{\text{ret}} dt \equiv \Delta p^{\text{inst}} + \Delta p_1^{\text{ret}}, \tag{12}$$

where

$$\Delta p^{\text{inst}} = -K \int \frac{dt}{R^2} = -K \int \frac{dR}{R^2}\left(\frac{dt}{dR}\right) \tag{13}$$

and

$$\Delta p_1^{\text{ret}} = -2K \int \beta_2 \frac{dt}{R^2} = -\frac{2Km}{m_2 c} \int \left(\frac{dR}{dt}\right) \frac{dt}{R^2}$$

$$= -\frac{2Km}{m_2 c} \int_{R(0)}^{R(t)} \frac{dR}{R^2} = -\frac{2Km}{m_2 c}\left[\frac{1}{R(0)} - \frac{1}{R(t)}\right], \tag{14}$$

where the relation: $\beta_2 = [m/(m_2 c)](dR/dt)$ has been used.

So, at first order in v_2 / c, Δp_1^{ret} depends only on the values of $R(0)$ and $R(t)$ being independent of the initial velocities of the objects.

The non-relativistic energy conservation equation for the instantaneous interaction [14]:

$$\frac{1}{2} m \left(\frac{dR}{dt}\right)^2 - \frac{K}{R} = E = \text{constant} \tag{15}$$

gives, together with (13):

$$\Delta p^{\text{inst}} = -K \sqrt{\frac{m}{2}} \int_{R(0)}^{R(t)} \frac{dR}{R^2 \sqrt{E + K/R}}. \tag{16}$$

This is a function of the constant *E* which, by considering the $R \to \infty$ limit, is seen to be equal to the asymptotic kinetic energy of the system T_∞.

To simplify the illustrative calculations which follow, it is convenient to consider the special case $E = T_\infty = 0$, in which case the solution for the initial value, v_0, of $v \equiv v_1 + v_2 = dR/dt$, which is given by:

$$\frac{1}{2}mv_0^2 = \frac{K}{R_0} \tag{17}$$

is the relative escape velocity of 1 and 2 for the instantaneous component of the forces:

$$v_{esc} = v_0 = \sqrt{\frac{2K}{mR_0}}. \tag{18}$$

In this case (16) simplifies to:

$$\Delta p^{inst}(R_{max}) = -\sqrt{\frac{Km}{2}}\int_{R(0)}^{R_{max}} \frac{dR}{R^{3/2}} = \sqrt{2Km}\left[\frac{1}{R(0)^{\frac{1}{2}}} - \frac{1}{R_{max}^{\frac{1}{2}}}\right] \tag{19}$$

and (14) gives:

$$\Delta p_1^{ret}(R_{max}) = -\frac{2Km}{m_2 c}\left[\frac{1}{R(0)} - \frac{1}{R_{max}}\right]. \tag{20}$$

As can be seen from Figure 1, the total retarded attractive forces (the sums of F^{inst} and F^{ret}) are larger, for the same spatial configuration, than instantaneous ones. Furthermore, the force acting on 2 is greater than that acting on 1 when $m_2 > m_1$. It follows that if the initial relative velocity is equal to v_{esc}, then the objects will not be brought to rest as $R \to \infty$, but instead 2 will first be brought to rest at a finite distance $R = R_{max}$ which is the solution of the equation $\Delta p^{inst} + \Delta p_2^{ret} = -p_0$, i.e.,

$$p_0 = K\int_{R_0}^{R_{max}} \frac{dR}{R^2}\left(\frac{dt}{dR}\right) + \frac{2Km}{m_1 c}\int_{R_0}^{R_{max}} \frac{dR}{R^2}, \tag{21}$$

where $p_0 = m_2 v_2(R_0) = m_1 v_1(R_0) = mv_{esc}$. Using (19), and (20) with exchange $1 \leftrightarrow 2$, it is found from (21) that $R_{max} = 1/u^2$, where u is the solution of the quadratic equation:

$$u^2 + \frac{m_1 c}{R_0^{\frac{1}{2}} p_0} u - \frac{1}{R_0} = 0 \tag{22}$$

which is

$$u = \left(\frac{1}{R_{\max}}\right)^{\frac{1}{2}} = \frac{m_1 c}{2R_0^{\frac{1}{2}} p_0}\left\{\left[1 + \left(\frac{2p_0}{m_1 c}\right)^2\right]^{\frac{1}{2}} - 1\right\}$$

$$= \frac{p_0}{R_0^{\frac{1}{2}} m_1 c} + O(\beta_{\text{esc}}^3), \tag{23}$$

where $\beta_{\text{esc}} \equiv v_{\text{esc}}/c$, so that

$$R_{\max} \simeq R_0\left(\frac{m_1 c}{p_0}\right)^2 = R_0\left(\frac{m_1}{m\beta_{\text{esc}}}\right)^2. \tag{24}$$

The momentum of 1 at the epoch that 2 comes to rest, $p_1(R_{\max})$ is given by a relation similar to (21):

$$p_1(R_{\max}) = p_0 + \Delta p_1(R_{\max})$$

$$= p_0 - \left[K\int_{R_0}^{R_{\max}} \frac{dR}{R^2}\left(\frac{dt}{dR}\right) + \frac{2Km}{m_2 c}\int_{R_0}^{R_{\max}} \frac{dR}{R^2}\right]. \tag{25}$$

Combining (19), (20) and (25) gives:

$$p_1(R_{\max}) = \frac{p_0^2}{c}\left[\frac{1}{m_1} - \frac{1}{m_2}\left(1 - \frac{p_0^2}{c^2 m_1^2}\right)\right]$$

$$= \frac{p_0^2}{mc}\left[\frac{m_2 - m_1}{m_1 + m_2}\right] + \frac{p_0^4}{c^3 m_1^2 m_2}. \tag{26}$$

Since 2 has vanishing momentum at $R = R_{\max}$, the fractional momentum imbalance:

$$\text{MI}(R_{\max}) \equiv [p_1(R_{\max}) - p_2(R_{\max})]/\, p_0$$

produced by the retarded forces for this configuration is:

$$\text{MI}(R_{\max}) = \frac{p_1(R_{\max})}{p_0} = \frac{p_0}{mc}\left[\frac{m_2 - m_1}{m_1 + m_2}\right] + \frac{p_0^3}{c^3 m_1^2 m_2}$$

$$= \beta_{\rm esc}\left[\frac{m_2 - m_1}{m_1 + m_2}\right] + O(\beta_{\rm esc}^3). \quad (27)$$

Following Laplace [2], the first numerical example considers a hypothetical relative motion of the Earth and the Moon. Instead of analysing the stability of the Keplerian orbit of the Moon around the Earth, a configuration for which it was shown [6] that retarded gravitational forces result in an unbalanced torque that would perturb the Moon's orbit, a collinear configuration, as shown in Figure 1, is considered. An initial separation equal to the actual mean Earth-Moon distance: $R_0 = 3.84 \times 10^8\,\text{m}$ is considered. The force constant K is $Gm_E m_M$, where m_E and m_M are the rest masses of the Earth and the Moon. Substituting $m_E = 5.98 \times 10^{27}\,\text{g}$, $m_M = 7.35 \times 10^{25}\,\text{g}$ [15] gives $v_{\rm esc} = 160.5\,\text{m/s}$ and $R_{\rm max} = 1.39 \times 10^{21}\,\text{m}$ or $1.47 \times 10^4\,\text{lyr}$. The quantity $\text{MI}(R_{\rm max})$ in Equation (27) measuring the breakdown of Newton's First Law takes the value: 5.22×10^{-7}. With $p_0 = mv_{\rm esc} = 1.17 \times 10^{28}\,\text{gms}^{-1}$, the unbalanced momentum of the system at $R = R_{\rm max}$ is $6.11 \times 10^{21}\,\text{gms}^{-1}$ corresponding to internal generation of $2.31 \times 10^{11}\,\text{MJ}$ of kinetic energy. For comparison, the total energy equivalent of the rest masses of the Earth and Moon is $5.44 \times 10^{31}\,\text{MJ}$. The velocity at $R = R_{\rm max}$ of the Earth-Moon system, initially at rest, is $1.01\,\mu\text{m/s}$ or $31.5\,\text{m}$ per century.

In the second example, a configuration of an electron and a proton with $m \simeq m_e$ initially separated by a Bohr radius: $R_0 = 5.3 \times 10^{-9}\,\text{cm}$ with relative velocity: $v_{\rm esc} = \sqrt{2q^2/m_e R_0} = 3.09 \times 10^6\,\text{m/sec}$, corresponding to an initial electron kinetic energy of $27.1\,\text{eV}$ is considered. The initial and final momenta of the electron are $5.26\,\text{keV}/\text{c}$ and $54.2\,\text{eV}/\text{c}$ for a maximum separation: $R_{\rm max} = 50\,\mu\text{m}$ and a final velocity of the ep system of $17.3\,\text{m/sec}$. The kinetic energy created by the unbalanced internal forces at $R = R_{\rm max}$ is $1.57 \times 10^{-6}\,\text{eV}$.

The work of Van Flandern [5] showed that the observed stability of the periods of binary pulsars requires that gravitational forces propagate at a speed much greater than that of light. Recent experimental work has also demonstrated the non-retarded nature of both electrical [11, 12] and magnetic [9, 10] force fields. The present work, discussing the simple and transparent case of collinear motion, has shown that retarded force fields necessarily violate Newton's laws of motion and, in consequence, the conservation of both energy and momentum. The calculated violations are, however, too small to set a meaningful limit on the force field speed. This was done by Van Flandern [5] who obtained a lower limit on the speed of gravitational forces of $2 \times 10^{10} c$. In spite of the above considerations, the paradigm of forces that always propagate at the speed of light, remains an essential postulate in both classical electrodynamics and general theory of relativity. Both theoretical and experimental evidences show that this postulate is now untenable.

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