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RELATIVITY OF OPTICAL ISOTROPY: PART I

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Abstract

It is proved here that an optical isotropic medium behaves as anisotropic when observed in uniform relative motion parallel to the direction of light propagation inside the medium, being the observed anisotropy incompatible with the known laws of optical crystallography.

1. Introduction

The refraction of light in moving mediums and, specially, the reflection of light by moving mirrors have been properly addressed by the special theory of relativity for the last century (see for instance: [1-6]). The main conclusion of these theoretical researches is that certain basic laws of optics have to be strongly modified. For example, if α is the angle of incidence and β the angle of reflection, the Second Law of the reflection of light, $\alpha = \beta$, becomes:

$$\cos\beta = \frac{-2(v/c)\cos\varphi + [1 + (v^2/c^2)\cos^2\varphi]\cos\alpha}{1 - 2(v/c)\cos\alpha\cos\varphi + (v^2/c^2)\cos^2\varphi},$$

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where φ represents the direction of relative motion with respect to the normal to the reflecting surface, and v is the relative velocity.

Here, I will also deal with an optical relativistic problem: the behavior of an optically isotropic medium at rest and in uniform relative motion, though the consequences go far beyond the complication of a formula. In this first part, the uniform relative motion will be parallel to the direction of light propagation inside the medium. The general case of any orientation for the relative motion with respect to the direction of light propagation will be addressed in the second part.

2. Discussion

In what follows, we will make use of a transparent rigid rod *R* of proper length L_o, made of an isotropic material *m* whose proper refractive index is n_o. The rod *R* is placed so that its longitudinal axis *S* is parallel to the X_o axis of its proper inertial reference frame RF_o. At both (the left and the right) ends of *R*, two emitting sources *A* and *B* emit photons in opposite directions along the *S* axis: S^+ and S^- directions, respectively (see Figure 1). Being n_o the refractive index of *m* and being *m* optically isotropic, light will travel across *R* at the same constant speed c/n_o in all directions.

At instant 0 in RF_o, the source A emits a photon $\tilde{a}$ in the S^+ direction and simultaneously *B* emits another photon $\tilde{b}$ in the S^- direction (see Figure 1). After a time t_o both photons collide at C_r (the geometrical center of *R*). Evidently, we will have:

$$t_o = \frac{(L_o/2)}{c/n_o} = \frac{n_o L_o}{2c}. \tag{1}$$

RF_v is another inertial reference frame whose spacetime diagram coincides with RF_o's at instant 0, and from whose perspective *R* moves at a uniform velocity $v = kc$, $0 < k < 1$, parallel to its X_v axis in the sense of the increasing *X* (Figure 2). From the point of view of RF_v, the following three measurements are possible:

(1) The velocity or *R* with respect to RF_v, which obviously is *v*.

(2) The velocities of $\tilde{a}$ and $\tilde{b}$ with respect to RF_v, which are the relativistic additions $(c/n_o + v)/(1 + v/cn_o)$ and $(c/n_o - v)/(1 - v/cn_o)$, respectively.

(3) The velocity c_{vam} of $\tilde{a}$ through *m* and the velocity c_{vbm} of $\tilde{b}$ through *m*.

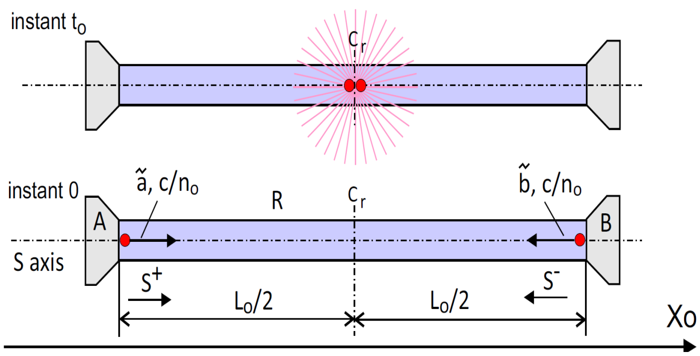

Figure 1.

The third type of velocities is ignored in the special theory of relativity, though it is plenty of physical meaning and can be calculated from RF_v: the distance inside *R* traversed by each photon divided by the time it takes for each photon to complete its journey. It represents the speed of light *with respect to the optical medium* observed in uniform relative motion. In both frames, RF_o and RF_v, the collision between $\tilde{a}$ and $\tilde{b}$ takes place at the center C_r of *R*, which means that in both frames it is observed that $\tilde{a}$ and $\tilde{b}$ traverse the same distance across *R*. In the case of RF_o that distance is $L_o/2$, while in the case of RF_v, and due to Lorentz contraction, that distance is $\gamma^{-1} L_o/2$. In RF_v, the photon $\tilde{a}$ begins to move at instant 0 and lasts a time t_{va} to reach C_r. Because of the phase difference in synchronization, the photon $\tilde{b}$ does not begin to move at instant 0, when $\tilde{a}$ does, but a time $t = \gamma v L_o/c^2$ after $\tilde{a}$ is emitted, and lasts a time t_{vb} to reach C_r. Therefore, in RF_v both photons begin to move at different instants and end their corresponding trips at the same instant. So $t_{va} \neq t_{vb}$, and taking into account that both photons travel through *m* the same distance $\gamma^{-1}L_o$, the photon $\tilde{a}$ and the photon $\tilde{b}$ will move through *m* at different velocities: c_{vam} for the photon $\tilde{a}$ and c_{vbm} for the photon $\tilde{b}$. We will now calculate these two velocities.

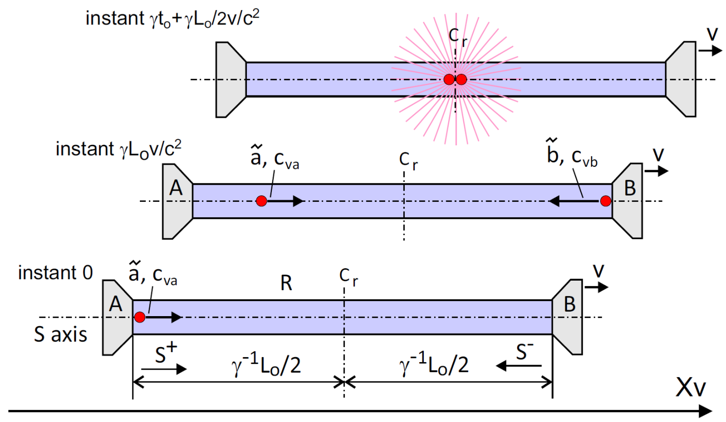

Figure 2.

Evidently $\tilde{a}$ moves with respect to RF_v for the same time t_{va} it does inside R. And the same applies to $\tilde{b}$ and t_{vb}. Since in RF_o it takes the same time t_o for both photons to complete their corresponding trips, in RF_v and for the case of $\tilde{a}$ we will have (according to Lorentz transformation):

$$c_{vam} = \frac{\gamma^{-1} L_o/2}{t_{va}} = \frac{\gamma^{-1} L_o/2}{\gamma t_o + \frac{\gamma v L_o/2}{c^2}}. \tag{2}$$

Now, taking into account that $t_o = (L_o/2)/(c/n_o)$, we can write:

$$c_{vh+} = \frac{\gamma^{-1} L_o/2}{\gamma t_o + \frac{\gamma v L_o/2}{c^2}} = \frac{\gamma^{-1} L_o/2}{\gamma \frac{L_o/2}{c/n_o} + \frac{\gamma v L_o/2}{c^2}}$$

$$= \frac{\gamma^{-1}}{\gamma\left(\frac{n_o}{c} + \frac{v}{c^2}\right)} = \frac{c^2}{\gamma^2(n_o c + v)}. \tag{3}$$

The refractive index n_{vs+} of m in the S^{+} direction is therefore:

$$n_{vs+} = \frac{c}{c_{vam}} = \frac{c}{c^2 / \gamma^2 (n_o c + v)} = \gamma^2 \left(n_o + \frac{v}{c} \right). \tag{4}$$

Or, taking into account that $v = kc$:

$$n_{vs+} = \frac{n_o + k}{1 - k^2}. \tag{5}$$

For the case of $\tilde{b}$ (and according again to Lorentz transformation) we will have:

$$c_{vbm} = \frac{\gamma^{-1} L_o/2}{t_{vb}} = \frac{\gamma^{-1} L_o/2}{\gamma t_o - \dfrac{\gamma v L_o/2}{c^2}}. \tag{6}$$

And the above line of calculation leads to:

$$n_{vs-} = \gamma^2 \left(n_o - \frac{v}{c} \right) = \frac{n_o - k}{1 - k^2}, \tag{7}$$

where n_{vs-} is the refractive index of m in the S^{-} direction. Figure 3 represents n_{vs+} and n_{vs-} in terms of n_o and the factor k of the relative velocity kc. Note that n_{vs+} is more sensitive to uniform relative motion than n_{vs-}, and that the difference between both refractive indexes only depends on k:

$$n_{vs+} - n_{vs-} = \frac{n_o + k}{1 - k^2} - \frac{n_o - k}{1 - k^2} = \frac{2k}{1 - k^2}. \tag{8}$$

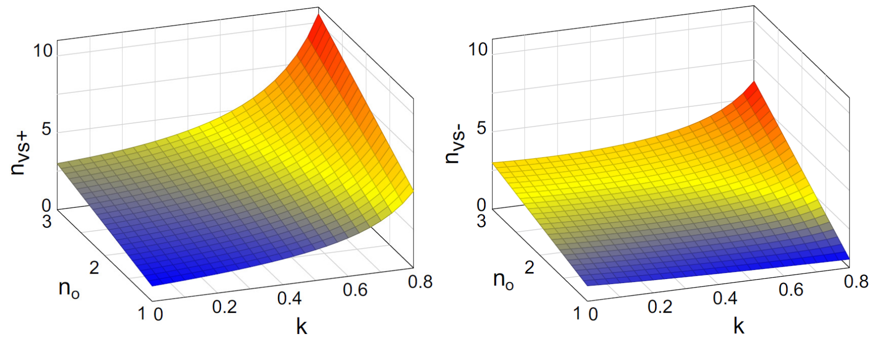

Figure 3.

3. Conclusions

The above short discussion proves that an optical isotropic medium behaves as anisotropic when observed in uniform relative motion, according to Lorentz transformation. And that the medium exhibits a type of anisotropy that is not compatible with the laws of optical crystallography: for a given direction *AB* inside an isotropic (or anisotropic) medium, light travels at the same speed in the sense from *A* to *B* as in the sense from *B* to *A*. As we have just proved, Lorentz transformation leads to the opposite conclusion. Note also that, as Figure 4 shows, that optical anisotropy is far from being infinitesimal, and increases exponentially with relative velocity. Being the speed of light through a transparent medium depending upon its magnetic permeability and its electric permittivity, it is hard to explain how these two electromagnetic constants can be different in both senses of the same direction.

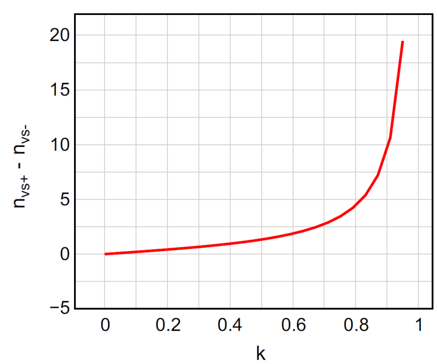

Figure 4.

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