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ORIGIN OF THE UNIVERSE FROM A BLACK HOLE: A THEORETICAL MODEL

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Abstract

A consistent theoretical model describing expansion of the universe from a central spherical uncharged Schwarzschild black hole is formulated with some modifications. The behaviour of the time dependent conformal factor, which is a temporal part of the geometric mass of the said black hole, is studied exhaustively in the different stages of the expansion. The concept of the dimensionless radius of the core black hole embedded by the space-time is emerged from this formulation. It is interpreted that the nature of such dimensionless radius plays a key role to govern the acceleration or deceleration of the universe. An inverse relation between the conformal factor and the scale factor evolves from the idea that the strength of the core black hole gets lowered to result an accelerated expansion of space-time. The nature of conformal factor and its effect towards the dynamics of the universe is established at different stages of expansion.

Keywords and phrases: expanding universe, Schwarzschild black hole, McVitte solution, geometric mass.

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1. Introduction

It is commonly believed that the universe started expanding from a big-bang singularity. Let us go with a little history of the development of cosmology. It was a milestone in the cosmological study when Einstein introduced the concept of general theory of relativity and proposed the Einstein equation. In 1922 Friedman [1] derived a set of equations, known as Friedman equation, which shows that the universe expands. At that time it contradicted the static model of the universe advocated by Einstein. In 1924, Hubble [2] measured the great distance to the nearest spiral nebulae which shows that these systems are indeed other galaxies. Hubble also proposed a correlation between distance and recession velocity, called Hubble's law. Lemaitre [3] derived Friedman equation independently and proposed that the inferred recession of the nebulae occurs since the universe expands. He also suggested that the expansion in forward time requires that the universe would contract backwards in time and continue until and unless it could bring all the mass into a single point, called a 'primordial atom', when and where the fabric time and space will come into existence. Alpher and Herman [4] predicted cosmic microwave background radiation (CMB) that favors big bang theory over steady state. Thus the concept of big bang theory came into existence. Lemaitre's big bang theory was later developed by Gamow. He and his collaborators introduced the concept of big bang nucleosynthesis [5]. Basically big bang theory depends on two major assumptions: the universality of physical law and cosmological principle.

In this paper, it is intended to study the expanding universe in terms of a Schwarzschild solution describing the gravitational field outside a spherical non-relativistic mass in absence of any electric charge as well as the cosmological constant. The motivation comes from the work of Arik and Senikoglu [6] who found an analytical exterior solution of Schwarzschild black hole in a cosmological space-time background. In 1933, McVittie [7] depicted a spherically symmetric metric describing a point mass embedded in an expanding spatially flat universe. Faraoni [9] and Jacques [8] analysed the solution of general relativity representing a black hole embedded in a cosmological background. Arakida [10] studied time delay in the Robertson McVitte space time. A number of authors [11, 12, 13, 14] exploited McVitte solution to study the exterior Schwarzschild solution with such scenario. In the present article, the work of Arik and Senikoglu [6] is extended by establishing a relation between the scale factor $a(t)$ and the conformal factor $b(t)$, and also indicating that such relation governs the dynamics of the universe. In the subsequent section a study is carried out in three different stages of the expansions in the scenario of the changing conformal factor $b(t)$ introduced by Arik and Senikoglu [6].

2. Formulation of the Model

In order to find a generalized solution for the expanding universe from a core black hole one may consider the metric as [6]

$$ds^2 = \frac{\left(1 - \frac{Gm}{2r}\right)^2}{\left(1 + \frac{Gm}{2r}\right)^2} dt^2 - a(t)^2 \left(1 + \frac{Gm}{2r}\right)^4 (dr^2 + r^2 d\Omega^2), \tag{1}$$

where, $a = a(t)$ is the so called scale factor representing the expansion of space at an instant t. It is to be noted that the Einstein tensor is not diagonalized here, rather the non-vanishing component indicating the momentum of the material body may be present.

One may assume that the universe begins to expand from a central dense primordial Schwarzschild black hole having radius $a(t)r$. As the cosmological principle reveals the universe is homogeneous as well as isotropic there is no harm to consider the central black hole as a spherically symmetric uncharged body. In the equation (1) the geometric mass m is not a constant at all, but depends on time. Since m varies with time the physical mass of the black hole would have changed with time at different phases of the universe. Now one may assume

$$m(t) = m_0 b(t), \tag{2}$$

where m_0 is the initial value but considered to be a dimensionless quantity. Arik and Senikoglu considered this $b(t)$ as a conformal factor whose significance at different phases is explored. The dimension of $b(t)$ is taken as the inverse length as that of the geometric mass. In the concerned work [6], the corresponding components of the Einstein tensors are calculated as

$$G_{00} = 3\frac{\dot{a}^2}{a^2}\frac{1}{\left(1-\frac{Gm_0 b(t)}{2r}\right)^2}\left(1+\frac{Gm_0}{2r}\left(b+2\dot{b}\frac{a}{\dot{a}}\right)\right)^2, \tag{3}$$

$$G_{01} = \frac{2Gm_0}{r^2\left(1-\frac{Gm_0 b(t)}{2r}\right)^2\left(1+\frac{Gm_0 b(t)}{2r}\right)^2}\frac{(b\dot{a}+\dot{b}a)}{a^2}, \tag{4}$$

$$G_{11} = G_{22} = G_{33} = -\frac{1}{\left(1-\frac{Gm_0 b(t)}{2r}\right)^3}\left[2\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{1}{a^2}\left(\frac{Gm_0}{2r}\right)(b\dot{a}^2+16a\dot{a}\dot{b}+2a\ddot{a}b+4a^2\ddot{b})\right.$$

$$+\frac{1}{a^2}\left(\frac{Gm_0}{2r}\right)^2(-b^2\dot{a}^2+4ab\dot{a}\dot{b}-2ab^2\ddot{a}+16a^2\dot{b}^2)$$

$$\left.+\frac{1}{a^2}\left(\frac{Gm_0}{2r}\right)^3(-b^3\dot{a}^2-12ab^2\dot{a}\dot{b}-2ab^3\ddot{a}-8a^2b\dot{b}^2-4a^2b^2\ddot{b})\right]. \tag{5}$$

As the black hole is embedded by the space-time, it must influence the space-time expansion. Therefore, an effective dimensionless radius $a(t)r$, which is supposed to play an important role to control the expansion of the universe is to be taken into account. That is the basic framework of the model to be discussed in this article. Since the universe is homogeneous and isotropic, followed by the cosmological principle, there is no harm to consider the black hole at the core as a spherically symmetric uncharged object having Schwarzschild radius. Now different phases of the universe in the framework of this formulation are to be studied.

2.1. Inflationary era

The inflation theory in cosmology was first theorized by Guth [15, 16], although the possibility of the existence of this cosmological phenomenon had been observed [36] much earlier. Later the Guth version of inflation theory was replaced by 'slow roll inflation' theory [18, 19, 21]. There are a number of models to address such cosmological inflation and a number of works [22-38] have been carried out in this arena. It has already been established that during the age of inflation the universe expanded rapidly, in the mathematical formulation approximately exponential. Such kind of expansion may be designed in terms of the model proposed by Arik and Senkoglu. During the inflationary era, the conformal factor $b(t)$ may be taken to be inversely proportional to the scale factor, i.e., $b(t) = \frac{1}{a(t)}$, which is consistent to the dimension of conformal factor. The idea behind such assumption is that the expansion of space may be resulted by the lowering of the conformal factor. It is to be shown that such relation leads to an approximated exponential expansion. Introducing the Hubble parameter $H = \frac{\dot{a}}{a}$, the non-zero diagonal components of the Einstein tensors take the form

$$G_{00} = 3H^2, \tag{6}$$

$$G_{11} = G_{22} = G_{33} = -3H^2 - 2\dot{H}\left(\frac{1 + \frac{Gm_0}{2ra(t)}}{1 - \frac{Gm_0}{2ra(t)}}\right). \tag{7}$$

In the above equation $\dot{H}$, a small quantity, represents the variation of Hubble parameter with time. To obtain its sign let us consider its expression in terms of the scale factor as follows.

$$\dot{H} = \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}. \tag{8}$$

Essentially the speed of expansion was more than its acceleration in the inflationary phase and therefore, the Hubble parameter would decrease in that era. As a result the second term in the equation (8) becomes negative, which results

$$G_{11} = G_{22} = G_{33} > -G_{00}. \tag{9}$$

In the inflationary era, it is often assumed $\dot{H} << H$, which leads to the approximate exponential expansion. In the inflation theory a scalar field $\phi(t)$, known as inflation, is supposed to play an important role, and in that phase it takes a value at which a large potential $V(\phi)$ becomes flat. Under the framework of said formulation, the change of such field $\phi(t)$ is related to the variation of the Hubble parameter as

$$\dot{H} = -\frac{1}{2}\left(\frac{1-\frac{Gm_0}{2ra}}{1+\frac{Gm_0}{2ra}}\right)\dot{\phi}^2. \tag{10}$$

The scalar field rolls very slowly down the potential, so that the Hubble parameter decreases slowly, and the universe experiences a more-or-less exponential inflation as formulated below.

$$a(t) = a_I \exp\left[\int_{t_I}^{t} H dt\right], \tag{11}$$

where t_I represents the instant at which the inflation begins and a_I is the value of a at t_I. Throughout the inflationary phase the dimensionless radius $a(t)r = 2Gm_0$ remains constant. On other side the geometric mass of the black hole decreases rapidly.

2.2. Radiation era

In the inflationary era, the conformal factor $b(t)$ continues to be decreased until it would reach to a constant value b_c. When it reaches to that value, the inflation comes to an end and the radiation age begins. Thus the conformal factor $b(t)$ and scale factor $a(t)$ become unrelated from the beginning of the radiation age and onwards. In this scenario the non-negative Einstein tensors may be calculated as

$$G_{00} = 3\frac{\dot{a}^2}{a^2}\frac{\left(1+\frac{Gm_0 b_c}{r}\right)^2}{\left(1-\frac{Gm_0 b_c}{r}\right)^2}, \tag{12}$$

$$G_{11} = G_{22} = G_{33} = -\left(2\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}\right)\frac{\left(1+\frac{Gm_0 b_c}{r}\right)^2}{\left(1-\frac{Gm_0 b_c}{r}\right)^2}, \tag{13}$$

$$G_{01} = \frac{2Gm_0}{r^2 b_c \left(1 + \frac{Gm_0 b_c}{r}\right)^2 \left(1 - \frac{Gm_0 b_c}{r}\right)^2} \frac{\dot{a}}{a^2}. \tag{14}$$

In the inflationary phase the term $\ddot{a}$ $\left(\text{with } \frac{\ddot{a}}{a} < \frac{\dot{a}^2}{a^2}\right)$ remains positive, but when the universe comes to the radiation age, $\ddot{a}$ becomes negative, signifying the universe gets decelerated. Followed by the equation (13) the pressure term becomes positive if $\left|\frac{\ddot{a}}{a}\right|$ exceeds the term $\frac{\dot{a}^2}{a^2}$. To satisfy the condition $p = \frac{\rho}{3}$, it is to be assumed $G_{11} = G_{22} = G_{33} = \frac{G_{00}}{3}$, which yields

$$a(t) \propto t^{\frac{1}{2}}. \tag{15}$$

This is well known behaviour of the scale factor when the universe is dominated by radiation. The dimensionless radius of the core black is no longer a constant, rather it becomes directly proportional to the scale factor $a(t)$. The geometric mass m_c becomes constant. Therefore, in that stage the dimensionless radius of the black hole embedded by the space time, given by $2m_c a(t)$, must increase with time.

Followed by the equation (14), it can easily be obtained that the non-diagonal momentum component G_{01} will be inversely proportional to $t^{\frac{3}{2}}$ and also be positive. The positivity of that momentum component signifies that the radiation comes outwards. This may support the fact that the radiation is created from the black hole.

2.3. Dust dominated era

In the matter dominated era, the stationary dust particles yields zero pressure. That is the key difference compared to the radiation dominated era. The conformal factor $b(t)$ no longer remains constant, although in that stage the $a(t)$ and $b(t)$ still remain unrelated. The relation $b(t) = e^{-\lambda t} b_c$, for small value of $-\lambda$ so that $|\dot{b} << b|$, signifies the conformal factor slowly decreases with time in the dust dominated era. The equations (13), (14) and (15) would also be valid in the matter dominated universe. Only mathematical difference is instead of the relation $p = \frac{\rho}{3}$, the condition $p = 0$ holds. Therefore

$$G_{11} = G_{22} = G_{33} = 0 \tag{16}$$

is to be imposed. Such condition in equation (13) makes it possible that

$$a(t) \propto t^{\frac{2}{3}}, \quad b(t) = 1. \tag{17}$$

The non-diagonal element G_{01} signifying the momentum component is found to vary as $\frac{1}{t^{5/3}}$ in the said period. The positivity of the momentum component indicates that the matter falls outward, but the slowly diminishing geometric mass $e^{-\lambda t} m_c$ of the core black hole indicates the matter is subsequently created in the universe.

2.4. Dark energy dominated era

In the scenario of dark energy dominated universe Arik and Senikoglu [6] took $b(t) = \frac{1}{a(t)}$, and shown $\rho = -p = 3H^2$, if the Hubble parameter H is taken as a constant therein. In the present model, the conformal factor $b(t) = \frac{1}{a(t)}$ is considered in the inflationary phase, i.e., in the early age of expansion when $a(t)$ would not been large enough. It is found that the universe in the present age is not only expanding, but it is also accelerating [39, 40]. Therefore, the present universe must be dominated by the dark energy although a small contribution of dust approximation may be taken into account. Also it is to be kept in our mind that in the universe at the present age or wherein the dark energy dominates over the matter and radiation the scale factor $a(t)$ is significantly high compared to that in the inflationary phase. Under the circumstances, the time dependent factor $b(t)$ may be taken as

$$b = \frac{1}{a + a_c}, \tag{18}$$

where $a_c = 1/bc$ and $a >> a_c$. In this scenario, the diagonal as well as non-diagonal components of the Einstein tensors are calculated and found to be

$$G_{00} = 3H^2 \left[\frac{1 - \left(1 - \frac{2a_c}{a}\right)\tau}{1 - \tau} \right]^2, \tag{19}$$

$$G_{01} = 4H \frac{\tau}{r(a - \tau)^2 (a + \tau)^2} \left(\frac{a_c}{a} \right) \left(1 - \frac{2a_c}{a} \right), \tag{20}$$

$$G_{11} = G_{22} = G_{33} = -3H^2 \left[1 - \frac{4}{3} \frac{\tau(1 - 5\tau)}{(1 - \tau)^2} \left(\frac{a_c}{a} \right) \right]$$
$$- 2\dot{H} \left[\frac{(1 + \tau)}{(1 - \tau)} - 2 \frac{\tau(1 + \tau^2)}{(1 - \tau)^3} \left(\frac{a_c}{a} \right) \right], \tag{21}$$

where

$$\tau = \frac{Gm_0}{2r(a + a_c)}. \tag{22}$$

The above set of relations are more generalized compared to that of Arik and Senikoglu [6] in the dark energy dominated regime. When the expansion of the universe is high enough, i.e., a_c is negligible relative to a and the Hubble parameter becomes more or less constant, the equations (19) and (21) can be approximated as $G_{11} = G_{22} = G_{33} = -3H^2 = G_{00}$. That is the result obtained by Arik and Senikoglu [6]. But their results differ from that of present article in case of non-diagonal component G_{01}. Unlike the paper of Arik and Senikoglu, here a non-zero positive term G_{01} is obtained as

$$G_{01} = 4H \frac{\tau}{r(1 - \tau)^2 (1 + \tau)^2} \left(\frac{a_c}{a} \right) \tag{23}$$

which indicates that 'something' flows outwards in the dark energy dominated universe. It may signify the creation of dark energy. The dimensionless radius $a(t)r$ takes the form $2m_0 \left[1 - \frac{a}{a_c} + O\left(\frac{a}{a_c} \right) \right] \simeq 2m_0$ (for large a). Therefore, in the present universe when the dark energy plays a dominant role the dimensionless radius of the black hole keeps its value constant as in the case of inflation, although the geometric mass is much less in the present age.

3. Discussion

The proposed model of the universe originated from the primordial black hole centered at its core removes the idea of the big-bang singularity, although the singularity at the origin of the black hole still persists. In the mathematical formulation, a conformal factor $b(t)$ introduced is nothing but a temporal part of geometric mass of the said black hole. Just before the beginning of expansion (at the moment of big bang) the whole space time entity is confined within a massive black hole in a dense form. Suddenly $b(t)$ and therefore, the geometric mass of the black hole starts decreasing. That indicates the strength of the black hole embedded by the space-time intends to be reduced. This phenomenon may be responsible for weakening the holding capacity of the space-time entity and consequently the big bang occurs. The behaviour of the $b(t)$ in the expanding scenario, i.e., when $\dot{a} > 0$ may be well observed if one studies Figure 1. In the phase of inflation (for $a \leq a_I$), the $b(t)$ falls down quite sharply. The inverse relation between the scale factor and conformal factor results an approximated exponential expansion and thus the inflation occurs. After that when the radiation age begins $(a > a_I)$, it becomes flat with the value $b = b_c$. In the radiation era, a deviation from that constant value is not observed at all. But in the matter dominated universe, a slight deviation from that value is evident, although it is not so remarkably high. In the dark energy dominated regime, Figure 1 clearly shows that the sharp descent of $b(t)$ is evident at the neighbourhood of $a = a_D$, i.e., at the beginning, but after that for a long run the conformal factor almost vanishes. It signifies that for a large scale of expansion, when the dark energy plays a dominant role, the geometric mass of the core black hole and hence its strength gets reduced to be nearly vanishing. But from Figure 2 as well as analytically it is found that the dimensionless radius remains at $2m_c$ in the dark energy dominated universe.

In the very early stage of expansion when the conformal factor $b(t)$ decreases, the dimensionless radius $a(t)r$ remains constant. In that stage, $b(t)$ varies inversely as $a(t)$. Under the circumstances it is shown that such inverse relation leads to an accelerated expansion so that the Hubble parameter varies slowly and the approximate nature of the exponential expansion (inflation) is resulted. In this stage, the dimensionless radius keeps its value at constant with the universe having an acceleration. When the conformal factor reaches to a minimum value, the inflation becomes halted and the radiation era begins. It has already been shown that in the radiation and matter dominated universe, $b(t)$ and $a(t)$ become unrelated to each other. Therein the dimensionless radius is directly proportional to $a(t)$ and the universe gets decelerated or in other words the acceleration of the universe is directed towards the center of the black hole and hence that of the universe. In the scenario when dark energy considerably dominates over matter again $b(t)$ becomes related to $a(t)$ by an inverse relation and again the acceleration of the universe is evident. Therefore, it can be confirmed that there must be a relation between the dimensionless radius of the space-time embedded black hole and nature of the expansion. It is thus found out that the dimensionless radius remains constant when the universe accelerates, but if such radius varies as $a(t)$, the deceleration in the expansion of the universe is evident. Therefore, it can be confirmed that there must be a relation between the dimensionless radius of the space-time embedded black hole and nature of the expansion. Thus the dynamics of expansion is controlled by primordial black hole at its central region.

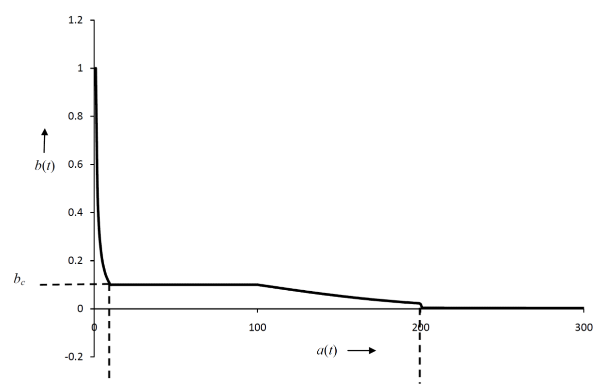

Figure 1. The relation between the conformal factor $b(t)$ and the scale

factor $a(t)$. $a = a_c$ represents the end of inflation and $a = a_D$ indicates the beginning of dark energy regime.

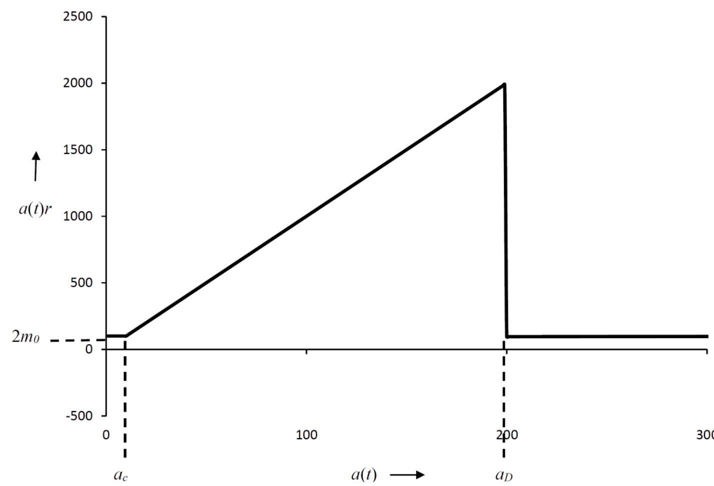

Figure 2. The relation between the dimensionless radius $a(t)r$ and the scale factor $a(t)$. $a = a_c$ and $a = a_D$ have same meanings as in Figure 1.

References

[1] A. A. Friedmann, Z. Physik 10 (1922), 377.

[2] E. Hubble, Proc. Nat. Acad. Sc. 15 (1929), 168.

[3] G. Lemaitre, Nature 128 (1931), 699.

[4] R. A. Alpher and H. Herman, Nature 162 (1948), 774.

[5] R. A. Alpher, H. Bethe and G. Gammow, Phys. Rev. 73 (1948), 803.

[6] M. Arik and Y. Senikoglu, Mod. Phys. Lett. A 29 (2014), 1450142; arXiv: 1401.4573 (2014).

[7] G. C. McVittie, Mon. Not. R. Astron. Soc. 93 (1933), 325.

[8] A. Jacques and V. Faraoni, Phys. Rev. D 76 (2007), 063510.

[9] V. Faraoni, Phys. Rev. D 80 (2009), 044013.

[10] H. Arakida, New Astron. 14 (2008), 264.

[11] M. Abdelqader and K. Lake, Phys. Rev. D 84 (2011), 044045.

[12] D. C. Guariento, A. M. da Silva and M. Foatanini, Phys. Rev. D 87 (2013), 064030.

[13] M. P. Hobson, R. Nandra and A. N. Lasenby, Mon. Not. R. Astron. Soc. 4 (2012), 2931.

[14] B. C. Nolan, Phys. Rev. D 58 (1998), 064000.

[15] A. Guth, The Inflationary Universe, Reading, Massachusetts, 1997.

[16] A. Guth, Phys. Rev. D 23 (1981), 347.

[17] A. A. Starobinsky, JETP Lett. 30 (1979), 682; Phys. Lett. B 91 (1980), 99.

[18] A. D. Linde, Phys. Lett. B 108 (1982), 389; 114 (1982), 431.

[19] A. D. Linde, Phys. Rev. Lett. 48 (1982), 335.

[20] A. D. Linde, Phys. Lett. B 129 (1983), 177.

[21] A. Albrecht and P. Steinhardt, Phys. Rev. Lett. 48 (1982), 1220.

[22] K. Freese, J. A. Frieman and A. V. Olinto, Phys. Rev. Lett. 65 (1990), 3233.

[23] F. C. Adams, J. R. Bond, K. Freese, J. A. Frieman and A. V. Olinto, Phys. Rev. D 47 (1993), 426; arXive: 9207245[hep-ph].

[24] K. Freese and W. H. Kinney, JCAP 1503 (2015), 044; arXiv:1403.5277 [astro-ph.CO].

[25] A. H. Chamseddine, V. Mukhanov and A. Vikman, JCAP 17 (2014), 1406; arXiv:1403.3961 [astro-ph.CO].

[26] R. Myrzakulov, L. Sebastiani and S. Vagnozzi, Eur. Phys. J. C 75 (2015), 444; arXiv:1504.07984 [gr-qc].

[27] R. Myrzakulov, L. Sebastiani, S. Vagnozzi and S. Zerbini, Fundamental J. Modern Phys. 8 (2015), 119; [arXiv:1505.03115 [gr-qc]].

[28] R. Myrzakulov, L. Sebastiani, S. Vagnozzi and S. Zerbini, arXiv:1510.02284 [gr-qc].

[29] L. Sebastiani and R. Myrzakulov, Int. J. Geom. Meth. Mod. Phys. 12 (2015), 1530003; arXiv:1506.05330 [gr-qc].

[30] E. Pajer and M. Peloso, Class. Quant. Grav. 30 (2013), 214002; arXiv:1305.3557 [hep-th].

[31] N. Barnaby and M. Peloso, Phys. Rev. Lett. 106 (2011), 181301; arXiv:1011.1500 [hepph].

[32] C. Armendariz-Picon, T. Damour and V. F. Mukhanov, Phys. Lett. B 458 (1999), 209; arXiv: 9904075[hep-th].

[33] G. R. Dvali and S. H. H. Tye, Phys. Lett. B 450 (1999), 72; arXiv:9812483[hep-ph].

[34] A. Sepehri, S. Capozziello and M. R. Setare, arXiv:1502.05493 [gr-qc].

[35] F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B 659 (2008), 703; [arXiv:0710.3755 [hep-th].

[36] A. A. Starobinsky, Phys. Lett. B 91 (1980), 99.

[37] S. Kaneda and S. V. Ketov, arXiv:1510.03524 [hep-th].

[38] L. Sebastiani, G. Cognola, R. Myrzakulov, S. D. Odintsov and S. Zerbini, Phys. Rev. D 89(2) (2014), 023518; arXiv:1311.0744 [gr-qc].

[39] A. G. Riess et al. (SNST Collabration), Astron. J. 116 (1998), 1009.

[40] S. Permutter et al. (SNST Collabration), Astron. J. 517 (1999), 565.