Fundamental Journal of Mathematical Physics
Vol. 3, Issue 2, 2013, Pages 55-62
Published online at http://www.frdint.com/

MATHEMATICAL FOUNDATIONS FOR APPROXIMATING PARTICLE BEHAVIOUR AT RADIUS OF THE PLANCK LENGTH

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Abstract

In this theoretical study, we have approximated particle behaviour at radius of the Planck Length, ℓ_p, such as, position, momentum and energy has different interpretation with the Heisenberg uncertainty. Heisenberg uncertainty principle said that we can not get the fixed measurement of both position and momentum simultaneously. On the other hand, this theoretical study found that we can measure position and momentum, or position and time simultaneously even when $\Delta y = 0$. But the problem then emerges from the last equation of this theory. Thus, we can not measure them if $\Delta y = 0$, because this measurement is bounded by $\Delta y \geq \ell_p \geq 0$.

1. Introduction

From many years ago, micro particle like electron is considered to have wave properties ([5-6]). The theoretical study also tries to prove why particle is considered to have wave properties. This is the last trilogy of my two previous papers for approximating particle behaviour at radius of the Planck Length.

Keywords and phrases: Planck length, Heisenberg uncertainty, microscopic system.

Received June 13, 2013

2. Mathematical Approximation

From [1] and we set $v_0 = 0$, $r = \ell_p$, we can rewrite one-dimensional equation for the particle that depends on time as follows:

$$y = -0.5\left(\frac{v^2 \cos(90 - \theta)}{r} + g \sin^2(90 - \theta)\right)t^2. \quad (1)$$

If we set $\theta = 0$ for all time t, then the coefficient of t^2 in (1) is an acceleration of gravity (g) of free fall on earth surface. But, it is not on earth surface, we are talking about a microscopic system.

From equation (1), we are trying to analyse particle behaviour at $r = \ell_p$. By using the same equation, we can analyse for $r << r_e$ in which r_e is earth radius ([2]).

Since (1) is derived from semicircular motion in [1], then we get the relation between r and θ as follows:

$$\cos\theta = \frac{x}{r},$$

$$\sin^2(90 - \theta) = \cos^2\theta = \left(\frac{x}{r}\right)^2. \quad (2)$$

In which x is a projection length of r toward horizontal axis. Since x is a projection length of r, thus $0 < x < r = \ell_p$. Here, our problem is difficult to determine the exact value of x because it is smaller than Planck length (ℓ_p). It is known that ℓ_p is measurable minimum length (see [4]). Thus, the interval value of (2) is $0 < \cos^2\theta < 1$. Now, we will investigate the cases around minimum or maximum of $\cos^2\theta$.

1. If we set $\cos^2\theta \sim 1$ (around the maximum), then equation (1) become free fall on earth surface, because $\cos(90 - \theta) = \sin\theta \sim 0$, therefore, we can approximate (1) with

$$y = -0.5gt^2.$$

But, it is impossible because it is not on earth surface.

2. If we set $\cos^2 \theta \sim 0$ (around the minimum), then (2) can be approximated by

$$\sin^2(90 - \theta) = \cos^2 \theta = \left(\frac{x}{r}\right)^2 \sim 0. \tag{3}$$

From (3), we rewrite (1)

$$y = -0.5\left(\frac{v^2 \cos(90 - \theta)}{r}\right)t^2,$$

$$= -0.5\left(\frac{v^2 t^2}{r}\right)\sin\theta = A(t)\sin\theta. \tag{4}$$

Equation (4) means that particle equation in (1) is a wave equation at very small radius.

By using trigonometry properties, we get from (3)

$$\sin^2 \theta \sim 1. \tag{5}$$

Equation (5) means

$$\sin\theta \sim 1 \tag{6}$$

or

$$\sin\theta \sim -1. \tag{7}$$

Since (6) and (7) are only approximation values, around 1 and –1, so we can rewrite (6) and (7) in the following interval:

$$-1 \le \sin\theta \le 1. \tag{8}$$

Multiplying (8) with $A(t)$ in (4), we have

$$-0.5\left(\frac{v^2}{r}\right)t^2 \le -0.5\left(\frac{v^2}{r}\right)t^2 \sin\theta \le 0.5\left(\frac{v^2}{r}\right)t^2,$$

$$y_{\min} = -0.5\left(\frac{v^2}{r}\right)t^2 \le y \le 0.5\left(\frac{v^2}{r}\right)t^2 = y_{\max}. \tag{9}$$

From (9), we interpret that the position of particle is in this interval. But, the most possible of finding particle from (9) is around the maximum or minimum of y.

Now, we will derive from (9) at $y = y_{\max}$, but we do not consider at $y = y_{\min}$ because at this point, kinetic energy will be negative.

$$y = 0.5\left(\frac{v^2}{r}\right)t^2, \tag{10}$$

$$\frac{dy}{dt} = v_y = \left(\frac{v^2}{r}\right)t, \tag{11}$$

$$\frac{dv_y}{dt} = a_y = \frac{v^2}{r}. \tag{12}$$

From (10) and (11) we have

$$y = 0.5 v_y t, \tag{13}$$

$$y = \frac{0.5 m v_y t}{m}, \tag{14}$$

$$y = \frac{0.5 p_y t}{m}. \tag{15}$$

From (15), Partial derivative with respect to momentum

$$\frac{\partial y}{\partial p_y} = \frac{\partial y}{\partial t} \cdot \frac{\partial t}{\partial p_y}, \tag{16}$$

$$= \left(\frac{v^2}{2r}\right)t \cdot \frac{\partial t}{m \partial v_y}, \tag{17}$$

$$= \left(\frac{v^2}{2r}\right)t \cdot \frac{1}{m(\partial v_y / \partial t)}, \tag{18}$$

$$= \left(\frac{v^2}{2r}\right)t \cdot \frac{1}{m(a_y)}. \tag{19}$$

Substituting (12) into (19)

$$= \left(\frac{v^2}{2r}\right)t \cdot \frac{r}{mv^2}, \tag{20}$$

$$\frac{\partial y}{\partial p_y} = \frac{t}{2m}, \tag{21}$$

$$\partial y = \frac{t}{2m}\partial p_y. \tag{22}$$

We can approximate (22)

$$\Delta y = \frac{t}{2m}\Delta p_y. \tag{23}$$

By using the same process, we have partial derivative for position with respect to time t

$$\frac{\partial y}{\partial t} = \frac{p_y}{2m},$$

$$\Delta y = \frac{p_y}{2m}\Delta t. \tag{24}$$

From (23) and (24), we reduce (9) into smaller interval as follows

$$y_{\min} - \Delta y \leq y \leq y_{\min} + \Delta y < 0, \quad 0 < y_{\max} - \Delta y \leq y \leq y_{\max} + \Delta y. \tag{25}$$

From (8) and (9), the probability of finding particle in the interval $y_{\max} + \Delta y$ or $y_{\min} - \Delta y$ is zero. From (11) we will calculate kinetic energy

$$v^2 = \frac{v_y r}{t}, \tag{26}$$

$$0.5mv^2 = \frac{mv_y r}{2t}, \tag{27}$$

$$E_k = \frac{mv_y r}{2t}, \tag{28}$$

$$E_k = \frac{p_y r}{2t}, \tag{29}$$

$$E_k t = \frac{p_y r}{2}. \tag{30}$$

We knew that the Planck length defined by

$$r = \ell_p = \sqrt{\frac{hG}{2\pi c^3}}. \tag{31}$$

Here h is Planck constant, G is the constant of universal gravitation, and c is the speed of light. Substituting (31) into (30), we have

$$E_k t = \frac{p_y}{2}\sqrt{\frac{hG}{2\pi c^3}}. \tag{32}$$

Substituting (23) to (32), we have

$$\Delta y = \frac{\Delta p_y p_y}{E_k 4m}\sqrt{\frac{hG}{2\pi c^3}}. \tag{33}$$

The boundary condition is $\Delta y \geq \ell_p$ [4], thus

$$\frac{\Delta p_y p_y}{E_k 4m} \geq 1. \tag{34}$$

Now, we will compare equations (23), (24), (25), (32), (33) and (34) with the Heisenberg uncertainty

Heisenberg Uncertainty (see [6])	**Arisetyawan**
For Position and momentum $\Delta y \Delta p_y \cong \frac{h}{2\pi}.$ For Energy and time $\Delta E \Delta t \cong \frac{h}{2\pi}$	Uncertainty for Position and momentum $\Delta y = \frac{t}{2m}\Delta p_y.$ Uncertainty for position and time $\Delta y = \frac{p_y}{2m}\Delta t.$ Probability interval of finding particle $0 < y_{\max} - \Delta y \leq y \leq y_{\max} + \Delta y,$ $y_{\min} - \Delta y \leq y \leq y_{\min} + \Delta y < 0.$ Relation between energy and time $E_k t = \frac{p_y}{2}\sqrt{\frac{hG}{2\pi c^3}}.$ Minimum condition for Δy $\Delta y = \frac{\Delta p_y p_y}{E_k 4m}\sqrt{\frac{hG}{2\pi c^3}}.$ With: $\frac{\Delta p_y p_y}{E_k 4m} \geq 1.$

3. Results and Discussion

If the position of particle can be measured exactly, $\Delta y = 0$. From Heisenberg uncertainty, the momentum will be unknown exactly, $\Delta p_y \sim \infty$. It means, we can not measure position and momentum simultaneously, and vice versa.

On the other hand, from (25), if $\Delta y = 0$, $y_{max} \leq y \leq y_{max}$, it means the position is certain $y = y_{max}$. By using (23) or (24), if $\Delta y = 0$, it means the momentum of particle or time are certain $(\Delta p_y = 0)$ or $(\Delta t = 0)$.

Unfortunately, the condition where momentum or position can be known exactly in the previous discussion is limited by equation (33) and (34) so that minimum Δy should be greater or equals than $\ell_p (\Delta y \geq \ell_p \geq 0)$. It causes the position, time and momentum of particle become a probability.

Equation (33) means, if we want to make Δy smaller than ℓ_p or equals to zero, we need very big (or infinite) kinetic energy. Equation (33) also means if we want to make Δy smaller than ℓ_p or equals to zero, the mass of particle should be very big (or infinite) too.

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