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INFINITELY MANY NEW CHARACTERIZATIONS OF T_i; $i = 0, 1, 2$, METRIZABLE, PSEUDOMETRIZABLE, R_i; $i = 0, 1$, AND T_0-IDENTIFICATION P PROPERTIES

CHARLES DORSETT

Department of Mathematics
Texas A&M University-Commerce
Commerce, Texas 75429
USA
e-mail: charles.dorsett@tamuc.edu

Abstract

Within this paper, T_0-identification spaces are used to give infinitely many new characterizations of T_0, T_1, T_2, metrizable, R_0, R_1, pseudometrizable, and T_0-identification P properties.

1. Introduction and Preliminaries

In 1936 [7], T_0-identification spaces were introduced and used to further characterize the metric property.

Definition 1.1. Let (X, T) be a space, let R be the equivalence relation on X defined by xRy iff $Cl(\{x\}) = Cl(\{y\})$, let X_0 be the set of R equivalence classes of X, let $N : X \to X_0$ be the natural map, and let $Q(X, T)$ be the decomposition topology on X_0 determined by (X, T) and the natural map N. Then

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$(X_0, Q(X, T))$ is the T_0-identification space of (X, T).

Theorem 1.1. *A space is pseudometrizable iff its T_0-identification space is metrizable.*

Within the 1936 paper [7], it was shown that for a space, its T_0-identification space is T_0. Further investigation of T_0-identification spaces [2] established that a space is T_0 iff it is homeomorphic to its T_0-identification space.

In 1975 [5], T_0-identification spaces and the R_1 separation axiom were used to further characterize T_2.

Theorem 1.2. *A space is R_1 iff its T_0-identification space is T_2.*

The R_1 separation axiom was introduced in 1961 [1] and used to further characterize T_2; and the R_0 separation axiom was revisited and used to further characterize T_1.

Definition 1.2. A space (X, T) is R_1 iff for x and y in X such that $Cl(\{x\}) \neq Cl(\{y\})$, there exist disjoint open sets U and V such that $x \in U$ and $y \in V$.

Definition 1.3. A space (X, T) is R_0 iff for each closed set C and each $x \notin C$, $C \cap Cl(\{x\}) = \phi$ [6].

Theorem 1.3. *A space is T_i iff it is $(T_{i-1}$ and $R_{i-1})$; $i = 1, 2$, respectively.*

The use of T_0-identification spaces in the characterizations of metrizable and T_2 raised the question of whether other topological properties could, in similar manner, be characterized using T_0-identification spaces, leading to the introduction and investigation of weakly Po spaces and properties [3].

Definition 1.4. Let P be a topological property for which $P\text{o} = (P$ and $T_0)$ exists. Then (X, T) is weakly Po iff its T_0-identification space $(X_0, Q(X, T))$ has property P. A topological property Qo for which weakly Qo exists is called a weakly Po property.

The two models motivating the definition of weakly Po spaces and properties are, in fact, weakly Po spaces and properties with weakly metrizable = weakly (pseudometrizable)o = pseudometrizable and weakly T_2 = weakly (R_1)o = R_1 [3]. Thus metrizable was the first known weakly Po property and T_2 was the second known weakly Po property.

Within the paper [3], it was shown that for a weakly Po property Qo, a space is weakly Qo iff its T_0-identification space is Qo; a space is weakly Qo iff its T_0-identification space is weakly Qo; and that T_1 is a weakly Po property with weakly T_1 = weakly (R_0)o = R_0. The fact that there are topological properties simultaneously shared by both a space and its T_0-identification space, as in the case of weakly Po, motivated the introduction and investigation of T_0-identification P properties [4].

Definition 1.5. Let S be a topological property. Then S is a T_0-identification P property iff both a space and its T_0-identification space simultaneously share property S.

Within the introductory T_0-identification P property paper [4], it was established that a topological property W is a T_0-identification P property iff W = weakly Wo.

Below established properties of T_0-identification spaces and T_0-identification P properties are used to give infinitely many new characterizations of T_0, T_1, T_2, metrizable, pseudometrizable, R_0, R_1, and T_0-identification P properties.

2. Infinitely Many New Characterizations of T_0

Definition 2.1. Let (X, T) be a topological space, let $(X_1, Q_1(X, T))$ be the T_0-identification space of (X, T), for each natural number $n \geq 2$, let $(X_n, Q_n(X, T))$ be the T_0-identification space of the space $(X_{n-1}, Q_{n-1}(X, T))$, and let $(\mathcal{X}, \mathcal{T}) = \{(X, T)\} \cup \{(X_n, Q_n(X, T)) \mid n \text{ is a natural number}\}$.

Within this paper, the notation given in Definition 2.1 will be repeatedly used.

Theorem 2.1. *Let* (X, T) *be a space. Then the following are equivalent*: (a) (X, T) *is* T_0, (b) *for each natural number* n, $(X_n, Q_n(X, T))$ *is homeomorphic to* (X, T), (c) *for some natural number* p, (X, T) *is homeomorphic to* $(X_p, Q_p(X, T))$, (d) *each element in* $(\mathcal{X}, \mathcal{T})$ *is* T_0 *and all elements of* $(\mathcal{X}, \mathcal{T})$ *are topologically equivalent*, (e) *each element of* $(\mathcal{X}, \mathcal{T})$ is T_0, (f) *all the elements of* $(\mathcal{X}, \mathcal{T})$ *are homeomorphic and*, *thus*, *all the elements of* $(\mathcal{X}, \mathcal{T})$ *are topologically equivalent*, *and* (g) *for a fixed natural number* p, $(X_p, Q_p(X, T))$ *is homeomorphic to* (X, T).

Proof. (a) implies (b): Since (X, T) is T_0, then, by the result above, (X, T) and $(X_1, Q_1(X, T))$ are homeomorphic. Thus the statement is true for $n = 1$. Assume the statement is true for $n = k$; k a natural number, $k \geq 1$. Since (X, T) is T_0 and (X, T) is homeomorphic to $(X_k, Q_k(X, T))$, then $(X_k, Q_k(X, T))$ is T_0, and by the result above, $(X_k, Q_k(X, T))$ is homeomorphic to its T_0-identification space $(X_{k+1}, Q_{k+1}(X, T))$. Hence (X, T) is homeomorphic to $(X_{k+1}, Q_{k+1}(X, T))$. Thus, if the statement is true for $n = k$, it is true for $n = k + 1$. Therefore, by mathematical induction, the statement is true for each natural number *n*.

Clearly, (b) implies (c).

(c) implies (d): Let *p* be a natural number such that (X, T) is homeomorphic to $(X_p, Q_p(X, T))$. If $p = 1$, then, by the result above, (X, T) is T_0. Thus consider the case that $p > 1$. Then $(X_{p-1}, Q_{p-1}(X, T))$ is a space and $(X_p, Q_p(X, T))$ is the T_0-identification space of $(X_{p-1}, Q_{p-1}(X, T))$, which implies $(X_p, Q_p(X, T))$ is T_0. Since T_0 is a topological property, (X, T) is T_0. Let *m* and *n* be natural numbers. Since (X, T) is T_0, then, by the argument above, $(X_m, Q_m(X, T))$ is homeomorphic to (X, T) and, since T_0 is a topological property, $(X_m, Q_m(X, T))$ is T_0. Thus each element of $(\mathcal{X}, \mathcal{T})$ is T_0 and is homeomorphic to (X, T). Hence, for the natural number n, $(X_n, Q_n(X, T))$ is homeomorphic to (X, T). Thus $(X_m, Q_m(X, T))$ is homeomorphic to (X, T) and (X, T) is homeomorphic to $(X_n, Q_n(X, T))$, which implies $(X_m, Q_m(X, T))$ and $(X_n, Q_n(X, T))$ are homeomorphic and hence topologically equivalent.

Clearly (d) implies (e).

(e) implies (f): Since $(X, T) \in (\mathcal{X}, \mathcal{T})$, then (X, T) is T_0 and, by the arguments above, all elements of $(\mathcal{X}, \mathcal{T})$ are topologically equivalent.

(f) implies (g): Since $(X_1, Q_1(X, T))$ is T_0 and in $(\mathcal{X}, \mathcal{T})$, $(X, T) \in (\mathcal{X}, \mathcal{T})$, and all elements of $(\mathcal{X}, \mathcal{T})$ are topologically equivalent, then (X, T) is T_0. Let p be a fixed natural number. Then, by the argument above, $(X_p, Q_p(X, T))$ is homeomorphic to (X, T).

(g) implies (a): Let p be a natural number such that $(X_p, Q_p(X, T))$ is homeomorphic to (X, T). Then for some natural number n, $(X_n, Q_n(X, T))$ is homeomorphic to (X, T) and, by the arguments above, (X, T) is T_0.

Theorem 2.2. *Let* (X, T) *be a space. Then each element of* $\{(X_n, Q_n(X, T)) | n \text{ is a natural number}\}$ *is* T_0 *and all elements of* $\{(X_n, Q_n(X, T)) | n$ *is a natural number*$\}$ *are topologically equivalent.*

Proof. *Let* $Y = X_1$ and let $S = Q_1(X, T)$ and for all $n \geq 1$, let $(Y_n, Q_n(Y, S)) = (X_{n+1}, Q_{n+1}(X, T))$. Then $(Y_1, Q_1(Y, S)) = (X_2, Q_2(X, T))$ is the T_0-identification space of $(X_1, Q_1(X, T)) = (Y, S)$, which is T_0, and for $n \geq 2$, $(Y_n, Q_n(Y, S)). = (X_{n+1}, Q_{n+1}(X, T))$ is the T_0-identification space of $(X_n, Q_n(X, T)) = (Y_{n-1}, Q_{n-1}(Y, S))$. Thus $\{(X_n, Q_n(X, T)) | n$ is a natural number$\}$ $= (\mathcal{Y}, \mathcal{S})$ and, by Theorem 2.1, for each $n \geq 1$, $(X_n, Q_n(X, T))$ is T_0 and all elements of $\{(X_n, Q_n(X, T)) | n$ is a natural number$\}$ are topologically equivalent.

3. Infinitely Many Characterizations of the Remaining Properties

Theorem 3.1. *Let* (X, T) *be a space and let* Q *be a* T_0*-identification* P *property. Then the following are equivalent*: (a) (X, T) *has property* Q, (b) *for each natural number* n, $(X_n, Q_n(X, T))$ *has property* Q *and all elements of* $\{(X_n, Q_n(X, T)) | n$ *is a natural number*$\}$ *are topologically equivalent*, (c) *for each natural number* n, $(X_n, Q_n(X, T))$ *has property* Q, (d) *for a fixed natural number* p, $(X_p, Q_p(X, T))$ *has property* Q, (e) *there exists a natural number* p *such that* $(X_p, Q_p(X, T))$ *has property* Q, (f) *all elements of* $(\mathcal{X}, \mathcal{T})$ *have property* Q, (g) *there is an element of* $(\mathcal{X}, \mathcal{T})$ *with property* Q, (h) *there exists a natural number* p *such that* $(X_p, Q_p(X, T))$ *has property* Qo, (i) *there exists a natural number* p *such that* $(X_p, Q_p(X, T))$ *has property* Qo *and all elements of* $\{(X_n, Q_n(X, T)) | n$ *is a natural number*$\}$ *are topologically equivalent*, (j) *for a fixed natural number* p, $(X_p, Q_p(X, T))$ *has property* Qo, (k) *for each natural number* n, $(X_n, Q_n(X, T))$ *has property weakly* Qo *and all elements of* $\{(X_n, Q_n(X, T)) | n$ *is a natural number*$\}$ *are topologically equivalent*, (l) *there exists a natural number* p *such that* $(X_p, Q_p(X, T))$ *is weakly* Qo *and all elements in* $\{(X_n, Q_n(X, T)) | n$ *is a natural number*$\}$ *are topologically equivalent*, (m) *there exists a natural number* p *such that* $(X_p, Q_p(X, T))$ *is weakly* Qo, (n) *for a fixed natural number* p, $(X_p, Q_p(X, T))$ *has property weakly* Qo, *and* (o) *all elements of* $(\mathcal{X}, \mathcal{T})$ *have property weakly* Qo.

Proof. (a) implies (b): Since Q is a T_0-identification P property and (X, T) has property Q, then $(X_1, Q_1(X, T))$ has property Q. By Theorem 2.2, all elements of $\{(X_n, Q_n(X, T)) | n$ is a natural number$\}$ are topologically equivalent and, since $(X_1, Q_1(X, T))$ has topological property Q, then all elements of $\{(X_n, Q_n(X, T)) | n$ is a natural number$\}$ have property Q.

Clearly (b) implies (c), (c) implies (d), and (d) implies (e).

(e) implies (f): Let p be a natural number such that $(X_p, Q_p(X, T))$ has property Q. By Theorem 2.2, all elements of $\{(X_n, Q_n(X, T)) | n$ is a natural number$\}$ are topologically equivalent and, since Q is a topological property, all elements of $\{(X_n, Q_n(X, T)) | n$ is a natural number$\}$ have property Q. Thus $(X_1, Q_1(X, T))$ has property Q and, since Q is a T_0-identification P property, (X, T) has property Q. Hence all elements of $(\mathcal{X}, \mathcal{T})$ have property Q.

Clearly (f) implies (g).

(g) implies (h): If (X, T) has property Q, then, since Q is a T_0-identification P property, $(X_1, Q_1(X, T))$ has property Q. Thus, in either case, there exists a natural number p such that $(X_p, Q_p(X, T))$ has property Q. Since $(X_p, Q_p(X, T))$ is T_0 and has property Q, then $(X_p, Q_p(X, T))$ has property Qo.

Clearly, by use of Theorem 2.2, (h) implies (i).

(i) implies (j): By Theorem 2.2, all elements of $\{(X_n, Q_n(X, T)) | n$ is a natural number$\}$ has property Qo. Thus, for a fixed natural number p, $(X_p, Q_p(X, T))$ has property Qo.

(j) implies (k): Let p be a fixed natural number such that $(X_p, Q_p(X, T))$ has property Qo. Since all the elements of $\{(X_n, Q_n(X, T)) | n$ is a natural number$\}$ are homeomorphic and Qo is a topological property, then $(X_1, Q_1(X, T))$ is Qo and, thus equivalently, Q. Since Q is a T_0-identification P property, then $Q =$ weakly Qo and $(X_1, Q_1(X, T))$ is weakly Qo, and, since weakly Qo is a topological property [3], for each natural number n, $(X_n, Q_n(X, T))$ has property weakly Qo.

Clearly (k) implies (l) and (l) implies (m).

(m) implies (n): Let p be a natural number such that $(X_p, Q_p(X, T))$ has property weakly Qo. Since, by Theorem 2.2, all elements of $\{(X_n, Q_n(X, T)) | n$ is a natural number$\}$ are homeomorphic, then all elements of $\{(X_n, Q_n(X, T)) | n$ is a natural number$\}$ are weakly Qo. Thus, for a fixed natural number q,

$(X_q, Q_q(X, T))$ has property weakly Qo.

(n) implies (o): Let p be a fixed natural number such that $(X_p, Q_p(X, T))$ has property weakly Qo. Since all elements of $\{(X_n, Q_n(X, T)) \mid n$ is a natural number are homeomorphic, then all elements of $\{(X_n, Q_n(X, T)) \mid n$ is a natural number$\}$ have property weakly Qo. Hence $(X_1, Q_1(X, T))$ is weakly Qo and, since Q is a T_0-identification property, then $Q =$ weakly Qo and $(X_1, Q_1(X, T))$ has property Q, which implies (X, T) has property $Q =$ weakly Qo and thus, all elements of $(\mathcal{X}, \mathcal{T})$ have property weakly Qo.

Clearly, since $Q =$ weakly Qo, (a) follows immediately from (o).

Combining Theorem 3.1 with the facts that R_0 = weakly (R_0)o = weakly T_1, R_1 = weakly (R_1)o = weakly T_2, and pseudometrizable = weakly (pseudometrizable)o = weakly (metrizable) are T_0-identification P properties gives the following infinite new characterizations for each of R_0, R_1, and pseudometrizable.

Corollary 3.1. *Let* (X, T) *be a space. Then the following are equivalent*: (a) (X, T) *has property* R_0, (b) *for each natural number* n, $(X_n, Q_n(X, T))$ *has property* R_0 *and all elements of* $\{(X_n, Q_n(X, T)) \mid n$ *is a natural number*$\}$ *are topologically equivalent*, (c) *for each natural number* n, $(X_n, Q_n(X, T))$ *has property* R_0, (d) *for a fixed natural number* p, $(X_p, Q_p(X, T))$ *has property* R_0, (e) *there exists a natural number p such that* $(X_p, Q_p(X, T))$ *has property* R_0, (f) *all elements of* $(\mathcal{X}, \mathcal{T})$ *have property* R_0, (g) *there is an element of* $(\mathcal{X}, \mathcal{T})$ *with property* R_0, (h) *there exists a natural number p such that* $(X_p, Q_p(X, T))$ *has property* (R_0)o $= T_1$, (i) *there exists a natural number p such that* $(X_p, Q_p(X, T))$ *has property* (R_0)o $= T_1$ *and all elements of* $\{(X_n, Q_n(X, T)) \mid n$ *is a natural number*$\}$ *are topologically equivalent*, (j) *for a fixed natural number* p, $(X_p, Q_p(X, T))$ *has property* (R_0)o $= T_1$, (k) *for each* *natural number* n, $(X_n, Q_n(X, T))$ *has property weakly* (R_0)o *and all elements of* $\{(X_n, Q_n(X, T)) \mid n$ *is a natural number*$\}$ *are topologically equivalent*, (l) *there exists a natural number* p *such that* $(X_p, Q_p(X, T))$ *is weakly* Qo *and all elements in* $\{(X_n, Q_n(X, T)) \mid n$ *is a natural number*$\}$ *are topologically equivalent*, (m) *there exists a natural number* p *such that* $(X_p, Q_p(X, T))$ *is weakly* (R_0)o, (n) *for a fixed natural number* p, $(X_p, Q_p(X, T))$ *has property weakly* (R_0)o, *and* (o) *all elements of* $(\mathcal{X}, \mathcal{T})$ *have property weakly* (R_0)o.

In like manner, replacing R_0 by R_1, T_1 by T_2, and weakly (R_0)o by weakly (R_1)o in Corollary 3.1 gives infinitely many new characterizations of R_1 and replacing R_0 by pseudometrizable, T_1 by metrizable, and weakly (R_0)o by weakly (pseudometrizable)o in Corollary 3.1 gives infinitely many new characterizations of pseudometrizable.

Theorem 3.2. *Let* (X, T) *be a space and let* Q *be a* T_0*-identification* P *property*. *Then the following are equivalent*: (a) (X, T) *has property* Qo, (b) *all the elements in* $(\mathcal{X}, \mathcal{T})$ *have property* Qo, (c) (X, T) *is* T_0 *and all elements in* $\{(X_n, Q_n(X, T)) \mid n$ *is a natural number*$\}$ *have property* Qo, (d) (X, T) *is* T_0 *and for each natural number* n, $(X_n, Q_n(X, T)))$ *has property* Q, (e) (X, T) *is* T_0 *and for each natural number* n, $(X_n, Q_n(X, T))$ *has property weakly* Qo, (f) (X, T) *is* T_0 *and there exists a natural number* p *such that* $(X_p, Q_p(X, T))$ *has property weakly* Qo, (g) (X, T) *is* T_0 *and for a fixed natural number* p, $(X_p, Q_p(X, T))$ *has property weakly* Qo, (h) (X, T) *is* T_0 *and for a fixed natural number* p, $(X_p, Q_p(X, T))$ *has property* Q, (i) (X, T) *has property* T_0 *and there exists a natural number* p *such that* $(X_p, Q_p(X, T))$ *has property* Q, (j) (X, T) is T_0 *and there exists a natural number* p *such that* $(X_p, Q_p(X, T))$ *has property* Qo, (k) (X, T) *has property* T_0 *and for a fixed natural number* p, $(X_p, Q_p(X, T))$ *has property* Qo, (l) (X, T) *has property* Q *and is homeomorphic to all elements of* $\{(X_n, Q_n(X, T)) \mid n$ *is a natural number*$\}$, (m)

(X, T) *has property Q and for a fixed natural number* p, (X, T) *is homeomorphic to* $(X_p, Q_p(X, T))$, (n) (X, T) *has property Q and there exists a natural number p such that* (X, T) *is homepmorphic to* $(X_p, Q_p(X, T))$, (o) (X, T) *has property weakly* Qo *and there exists a natural number p such that* (X, T) *is homeomorphic to* $(X_p, Q_p(X, T))$, (p) (X, T) *has property weakly* Qo *and for a fixed natural number* p, (X, T) *is homeomorphic to* $(X_p, Q_p(X, T))$, *and* (q) (X, T) *is weakly* Qo *and is topologically equivalent to all the elements of* $\{(X_n, Q_n(X, T)) | n$ *is a natural number*$\}$.

Proof. (a) implies (b): Since (X, T) is T_0, then all elements of $(\mathcal{X}, \mathcal{T})$ are topologically equivalent and, since (X, T) has property Qo, then all elements of $(\mathcal{X}, \mathcal{T})$ have property Qo.

(b) implies (c): Since $(X, T) \in (\mathcal{X}, \mathcal{T})$, then (X, T) is T_0 and, since $\{(X_n, Q_n(X, T)) | n$ is a natural number$\} \subseteq (\mathcal{X}, \mathcal{T})$, then the remainder of the statement is true.

Since Qo implies Q, then (d) follows immediately from (c), since Q is a T_0-identification P property, $Q =$ weakly Qo and (e) follows immediately from (d), and clearly, (f) follows immediately from (e).

(f) implies (g): Let p be a fixed natural number. Since all elements of $\{(X_n, Q_n(X, T)) | n$ is a natural number$\}$ are topologically equivalent and there exists a natural number q such that $(X_q, Q_q(X, T))$ has property weakly Qo, then $(X_p, Q_p(X, T))$ has property weakly Qo.

Since $Q =$ weakly Qo, then (h) follows immediately from (g) and, clearly, (i) follows immediately from (h).

(i) implies (j): Let p be a natural number such that $(X_p, Q_p(X, T))$ has property Q. Since $(X_p, Q_p(X, T))$ is T_0, then $(X_p, Q_p(X, T))$ has property Qo.

(j) implies (k): Since all elements of $\{(X_n, Q_n(X, T)) | n$ is a natural number$\}$

are topologically equivalent and there exists a natural number q such that $(X_q, Q_q(X, T))$ is Qo, then for a fixed natural number p, $(X_p, Q_p(X, T))$ has property Qo.

(k) implies (l): Since (X, T) is T_0, then all elements of $(\mathcal{X}, \mathcal{T})$ are topologically equivalent and since for a fixed natural number p, $(X_p, Q_p(X, T))$ has property Qo, then (X, T) has property Qo, which implies (X, T) has property Q.

Clearly, (l) implies (m) and (m) implies (n).

Since Q is a T_0-identification P property, $Q =$ weakly Qo and (o) follows immediately from (n).

(o) implies (p): Let q be a natural number such that (X, T) is homeomorphic to $(X_q, Q_q(X, T))$. Let p be a fixed natural number. Since all element of $\{(X_n, Q_n(X, T)) | n$ is a natural number$\}$ are topologically equivalent, then $(X_q, Q_q(X, T))$ and $(X_p, Q_p(X, T))$ are homeomorphic. Hence, (X, T) is homeomorphic to $(X_p, Q_p(X, T))$.

(p) implies (q): Since 1 is a fixed natural number, (X, T) is homeomorphic to $(X_1, Q_1(X, T))$. Let p be a natural number. Since all elements of $\{(X_n, Q_n(X, T)) | n$ is a natural number$\}$ are topologically equivalent, then $(X_1, Q_1(X, T))$ is homeomorphic to $(X_p, Q_p(X, T))$ and, thus, (X, T) and $(X_p, Q_p(X, T))$ are topologically equivalent.

(q) implies (a): Since $Q =$ weakly Qo, then (X, T) has property Q and since (X, T) is homeomorphic to $(X_1, Q_1(X, T))$, which is T_0, then (X, T) has property Qo.

Since R_0 is a T_0-identification P property and $(R_0)\text{o} = T_1$, the next result follows immediately from Theorem 3.2.

Corollary 3.2. *Let* (X, T) *be a space. Then the following are equivalent*: (a)

(X, T) *is* T_1, (b) *all elements of* $(\mathcal{X}, \mathcal{T})$ *are* T_1, (c) (X, T) *is* T_0 *and all elements of* $\{(X_n, Q_n(X, T)) \mid n$ *is a natural number*$\}$ *has property* T_1, (d) (X, T) *is* T_0 *and for each natural number* n, $(X_n, Q_n(X, T))$ *has property* R_0, (e) (X, T) is T_0 *and for each natural number* n, $(X_n, Q_n(X, T))$ *has property weakly* (R_0)o, (f) (X, T) *is* T_0 *and there exists a natural number p such that* $(X_p, Q_p(X, T))$ *has property weakly* (R_0)o, (g) (X, T) *is* T_0 *and for a fixed natural number* p, $(X_p, Q_p(X, T))$ *has property weakly* (R_0)o, (h) (X, T) *is* T_0 *and for a fixed natural number* p, $(X_p, Q_p(X, T))$ *has property* R_0, (i) (X, T) *is* T_0 *and there exists a natural number p such that* $(X_p, Q_p(X, T))$ *has property* R_0, (j) (X, T) *is* T_0 *and there exists a natural number p such that* $(X_p, Q_p(X, T))$ *has property* T_1, (k) (X, T) *is* T_0 *and for a fixed natural number* p, $(X_p, Q_p(X, T))$ *has property* T_1, (l) (X, T) *is* R_0 *and is homeomorphic to all elements of* $\{(X_n, Q_n(X, T)) \mid n$ *is a natural number*$\}$, (m) (X, T) *has property* R_0 *and for a fixed natural number* p, (X, T) *is homeomorphic to* $(X_p, Q_p(X, T))$, (n) (X, T) *has property* R_0 *and there exists a natural number p such that* (X, T) *is homeomorphic to* $(X_p, Q_p(X, T))$, (o) (X, T) *has property weakly* (R_0)o *and there exists a natural number p such that* (X, T) *is homeomorphic to* $(X_p, Q_p(X, T))$, (p) (X, T) *has property weakly* (R_0)o *and for a fixed natural number* p, (X, T) *is homeomorphic to* $(X_p, Q_p(X, T))$, *and* (q) (X, T) *has property weakly* (R_0)o *and is topologically equivalent to all the elements of* $\{(X_n, Q_n(X, T)) \mid n$ *is a natural number*$\}$.

In a similar manner, using the fact that R_1 is a T_0-identification *P* property and $R_1 =$ weakly (R_1)o $=$ weakly T_2, Theorem 3.2 can be used to give infinitely many new characterizations of T_2. Additionally, infinitely many more new characterizations of T_2 can be given by replacing each statement "(X, T) is T_0" by "(X, T) is T_1".

Likewise, the fact that pseudometrizable is a T_0-identification *P* property and pseudometrizable = weakly (pseudometrizable)o = weakly (metrizable) can be used to give infinitely many more characterizations of metrizable. Infinitely many more characterizations of metrizable can be obtained by replacing each statement "(X, T) is T_0" by "(X, T) is T_1", then by "(X, T) is T_2", and any statement "(X, T) has property P", where P is any other property stronger than T_0 and weaker than metrizable.

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