Fundamental Journal of Modern Physics
ISSN: 2249-9768
Volume 24, Issue 2, 2025, Pages 77-82
This paper is available online at http://www.frdint.com/
Published online December 25, 2025

CASIMIR WORMHOLES IN $f(Q)$ MODIFIED GRAVITY

PETER K. F. KUHFITTIG

Department of Mathematics
Milwaukee School of Engineering
Milwaukee, Wisconsin 53202-3109
USA
e-mail: kuhfitti@msoe.edu

Abstract

It is well known that a Morris-Thorne wormhole can only be held open by violating the null energy condition. This violation calls for the existence of so-called exotic matter, as exemplified by the Casimir effect. Being a rather small effect, it needs to be boosted by assuming an appropriate modification of the underlying gravitational theory.

1. Introduction

Wormholes are handles or tunnels in spacetime that connect widely separated regions of our Universe or different universes altogether. Even though wormholes are just as good a prediction of Einstein's theory as black holes, they are subject to severe restrictions from quantum field theory. More precisely, holding a wormhole open requires a violation of

Keywords and phrases: Morris-Thorne wormholes, $f(Q)$ gravity, Casimir effect.

Received December 14, 2025; Accepted December 18, 2025

the null energy condition, calling for the existence of "exotic matter" [1]. This violation is more of a practical than conceptual problem, as illustrated by the Casimir effect [2]: exotic matter can be made in the laboratory. Being a rather small effect, it is not immediately obvious that it is sufficient for supporting a traversable wormhole.

2. Morris-Thorne Wormholes

In this section, we are going to recall that wormholes are handles or tunnels in spacetime that connect widely separated regions of our Universe or different universes altogether. The first detailed analysis of humanly traversable wormholes was carried out by Morris and Thorne [1]. With the Schwarzschild solution in mind, they proposed the following static and spherically symmetric line element to model a wormhole spacetime:

$$ds^2 = -e^{2\Phi(r)}dt^2 + \frac{dr^2}{1-\frac{b(r)}{r}} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \qquad (1)$$

using units in which $c = G = 1$. Here $\Phi = \Phi(r)$ is usually referred to as the *redshift function*, which must be finite everywhere to prevent the occurrence of an event horizon. The function $b = b(r)$ is called the *shape function* since it determines the spatial shape of the wormhole when viewed, for example, in an embedding diagram. The spherical surface $r = r_0$ is called the *throat* of the wormhole. According to Ref. [1], at the throat, $b = b(r)$ must satisfy the following conditions: $b(r_0) = r_0$, $b(r) < r$ for $r > r_0$, and $b'(r_0) \leq 1$, called the *flare-out condition*. This condition can only be satisfied by violating the null energy condition (NEC), which states that

$$T_{\alpha\beta}k^{\alpha}k^{\beta} \geq 0 \qquad (2)$$

for all null vectors k^{α}, where $T_{\alpha\beta}$ is the energy momentum tensor. As noted above, matter that violates the NEC is called "exotic" in Ref. [1]. In particular, for the outgoing null vector $(1, 1, 0, 0)$, the violation becomes $T_{\alpha\beta}k^{\alpha}k^{\beta} = \rho + p_r < 0$. Here, $T^t{}_t = -\rho(r)$ is the energy density, $T^r{}_r = p_r(r)$ is the radial pressure, and $T^{\theta}{}_{\theta} = T^{\phi}{}_{\phi} = p_t(r)$ is the lateral (transverse) pressure. Another requirement is asymptotic flatness: $\lim_{r\to\infty}\Phi(r) = 0$ and $\lim_{r\to\infty} b(r)/r = 0$.

For later reference, we now list the Einstein field equations:

$$\rho(r) = \frac{1}{8\pi}\frac{b'}{r^2}, \tag{3}$$

$$p_r(r) = \frac{1}{8\pi}\left[-\frac{b}{r^3} + 2\left(1 - \frac{b}{r}\right)\frac{\Phi'}{r}\right], \tag{4}$$

and

$$p_t(r) = \frac{1}{8\pi}\left(1 - \frac{b}{r}\right)\left[\Phi'' - \frac{b'r - b}{2r(r-b)}\Phi' + (\Phi')^2 + \frac{\Phi'}{r} - \frac{b'r - b}{2r^2(r-b)}\right]. \tag{5}$$

Since we are primarily interested in wormholes, we are going to follow Ref. [3], which uses a simple but commonly used form of $f(Q)$: $f(Q) = \alpha Q + \beta$ [3], where α and β are free parameters. Since $f'(Q)$ is a constant and $f''(Q) = 0$, this produces the Einstein field equations with a cosmological constant [4]. The corresponding field equations are [4]

$$\rho = \frac{\alpha b'}{r^2} + \frac{\beta}{2}, \tag{6}$$

$$p_r = \frac{1}{r^3}\left[2\alpha r(r-b)\Phi' - \alpha b\right] - \frac{\beta}{2}, \tag{7}$$

and

$$p_t = \frac{1}{2r^3}\left[\alpha(r\Phi' + 1)(-rb' + 2r(r-b)\Phi' + b)\right] + \frac{\alpha(r-b)\Phi''}{r} - \frac{\beta}{2}. \tag{8}$$

3. Casimir Wormholes

Attempts to overcome the theoretical and practical problems confronting Morris-Thorne wormholes have relied heavily on various modified gravitational theories. A recently proposed modified theory, called $f(Q)$ gravity, is due to Jimenez, et al. [4]. Here Q is the non-metricity scalar from the field of differential geometry. The action for this gravitational theory is given by

$$S = \int \frac{1}{2} f(Q)\sqrt{-g}\, d^4x + \int \mathcal{L}_m \sqrt{-g}\, dx^4, \tag{9}$$

where $f(Q)$ is an arbitrary function of Q, $\mathcal{L}_m$ is the Lagrangian density of matter, and g is the determinant of the metric tensor $g_{\mu\nu}$. Even though it is a fairly new theory, numerous applications have already been found; see, for example [5, 6, 7, 8, 9, 10].

The *Casimir effect* appears between two parallel, closely spaced, uncharged metallic plates in a vacuum. It was predicted in 1948 and subsequently confirmed. The energy density is given by

$$\rho(r) = \frac{\hbar c \pi^2}{720 r^4}. \tag{10}$$

This can now be combined with Eq. (6) to yield

$$\frac{\hbar c \pi^2}{720 r^4} = \alpha \frac{b'(r)}{r^2} + \frac{\beta}{2}. \tag{11}$$

Solving for $b'(r)$ and integrating, we obtain

$$b(r) = \frac{1}{\alpha}\left[-\frac{\hbar c \pi^2}{720 r}\frac{1}{r} - \frac{\beta}{2}\frac{r^3}{3}\right] + c. \tag{12}$$

The final form is

$$b(r) = r_0 + \frac{1}{\alpha}\left[\frac{\hbar c\pi^2}{720}\frac{r - r_0}{r_0 r} - \frac{\beta}{6}(r - r_0)(r_0^2 - r_0 r + r^2)\right]. \tag{13}$$

Observe that $b(r_0) = r_0$, as required.

Given that

$$m(r) = \int_{r_0}^{r} \rho(r')4\pi(r')^2 dr' = \frac{1}{2}b(r) \tag{14}$$

from Eq. (3), it now follows from Eq. (13) that thanks to the free parameter α from $f(Q)$ gravity, the mass of the wormhole can be macroscopic. This is our main conclusion.

4. Conclusion

The purpose of this paper is to present a new wormhole solution by means of a recently proposed modified gravitational theory called $f(Q)$ modified gravity. Here Q is the nonmetricity scalar from differential geometry leading to the Casimir effect. Initially a small effect, it leads to the shape function, Eq. (13). It follows that thanks to the free parameter α from $f(Q)$ modified gravity, the mass of the wormhole can be macroscopic.

References

[1] M. S. Morris and K. S. Thorne, Wormholes in spacetime and their use for interstellar travel: A tool for teaching general relativity, American Journal of Physics 56 (1988), 395-412.

[2] H. G. B. Casimir, On the attraction between two perfectly conducting plates, Proceedings of the Royal Netherlands Academy of Arts and Sciences 51 (1948), 793-795.

[3] Z. Hassan, S. Mandal and P. K. Sahoo, Traversable wormhole geometries in $f(Q)$

gravity, Fortschritte der Physik 69 (2021), ID: 2100023.

[4] J. B. Jimenez, L. Heisenberg and T. Koivisto, Coincident general relativity, Physical Review D 98 (2018), ID: 044048.

[5] G. Mustafa, Z. Hassan, P. H. R. S. Moraes and P. K. Sahoo, Wormhole solutions in symmetric teleparallel gravity, Physics Letters B 821 (2021), ID: 136612.

[6] U. K. Sharma, Shweta and A. K. Mishra, Traversable wormhole solutions with non-exotic fluid in framework of $f(Q)$ gravity, International Journal of Geometric Methods in Modern Physics 19 (2021), ID: 2250019.

[7] A. Banerjee, A. Pradhan, T. Tangphati and F. Rahaman, Wormhole geometries in $f(Q)$ gravity and the energy conditions, European Physical Journal C 81 (2021), ID: 1031.

[8] G. Mustafa, Z. Hassan and P. K. Sahoo, Traversable wormholes inspired by non-commutative geometries in $f(Q)$ gravity with conformal symmetry, Annals of Physics 437 (2022), ID: 168751.

[9] F. Parsaei, S. Rastgoo and P. K. Sahoo, Wormhole in $f(Q)$ gravity, European Physical Journal Plus 137 (2022), ID: 1083.

[10] Z. Hassan, G. Mustafa, Joao R. L. Santos and P. K. Sahoo, Embedding procedure and wormhole solutions in $f(Q)$ gravity, Europhysics Letters 139 (2022), ID: 39001.