

CAN MASSLESS NEUTRINOS OSCILLATE IN PRESENCE OF MATTER?

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Abstract

In presence of matter the possibility of flavor oscillation of massless neutrino is explored. A comparison between vacuum oscillation and the matter induced oscillation of massless neutrinos is carried out to examine whether the flavor transition is possible in the framework of standard model if there is background matter. The Stodolsky type of equation describing the neutrino oscillation phenomenon as the motion of a spherical pendulum in flavor space is deduced. That pendular model is studied with zero vacuum oscillation frequency implying the zero neutrino mass evolved in the framework of standard model, but non-zero frequency arising due to the MSW effect. The implication of the non-zero term present in the Stodolsky equation at zero vacuum frequency is addressed properly.

Introduction

According to the standard model of electroweak theory [1] the neutrino is considered as massless, although a bare minimum extension of this model can generate the neutrino mass, however, it is still questionable why the neutrino mass is so small. In 1957 Pontecorvo [2] proposed the concept of neutrino oscillation. That is

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nothing but a quantum mechanical phenomenon in which one type of neutrino flavor can later be measured to have another kind. Maki et al. [3] proposed $\nu_e - \nu_\mu$ mixing as well as virtual $\nu_e - \nu_\mu$ transmutation. The flavor eigenstates of neutrinos are the coherent mixtures of their mass eigenstates and thus the concept of neutrino mass is evolved in the perspective of neutrino oscillation. This type of neutrino oscillation is considered to be the vacuum oscillation which can successfully explain the ‘atmospheric neutrino anomaly’; but the vacuum oscillation fails to explain the solar neutrino fluxes observed from the various experimental data. Therefore, the matter effect must be taken into account while studying the solar neutrino fluxes as the resonant oscillation of neutrinos occurs in the solar matter. In 1978 Wolfenstein [4] introduced the concept of neutrino oscillation in presence of matter, but unfortunately he used wrong sign of matter profile. Later, this matter effect on neutrino oscillation was developed by Mikheev and Smirnov [5] to introduce the resonance phenomenon in neutrino oscillation. Thus the influence of background matter on neutrino oscillation is also known as MSW effect. In explaining the solar neutrino data the MSW effect must be taken into account. In matter background the electron type of neutrinos interact with electrons in the matter, while the interaction effect of the muon (or tau) neutrinos to the muons (or taus) is ignorable in the energy range of solar neutrinos. As a result the nature of oscillation of electron neutrino is quite different from that of other two flavors and that is the key factor for introducing the MSW effect. Eventually, the MSW effect modifies the neutrino oscillation phenomenon. Now the question may arise whether such effect can generate the flavor transitions even in the framework of Standard Model.

Wolfenstein claimed to show that if all neutrinos are massless it is possible to have oscillations occur when neutrinos pass through the matter. We shall examine whether such demand has any justification. It is to be remembered that the existence of neutrino mass is an ad-hoc assumption adopted to explain the neutrino oscillation phenomenon. In other words, if the flavor transitions occur the neutrinos must have the non-zero mass. But does the converse hold, i.e., can an effective mass generate the neutrino oscillation? Which factor is then responsible for the oscillation phenomenon? We must address such questions arose. But at first for simplicity one can assume the neutrino oscillation occurs only between two flavors: electron neutrino and ℓ neutrino (which is a mixture of μ and τ neutrinos), whereas the initial beam consists of purely electron neutrino flavors. In Section 2, we shall examine the feasibility of matter induced oscillation in absence of any neutrino rest mass.

Mikheev and Smirnov could realized that the presence of non-uniform background matter would produce the density dependence effective mass. We shall verify whether such effective mass can generate the neutrino oscillation in the framework of standard model. In Section 3, we shall introduce Stodolsky type of equation representing the dynamics of the neutrino oscillation and study what information it can give about the flavor transitions with zero rest mass of neutrinos, i.e., in the framework of standard model.

2. MSW Effect in the Framework of Standard Model

In the scenario of flavor transition it is known that the flavor eigenstates are the superpositions of the mass eigenstates with the vacuum mixing angle θ_0 giving the information about the nature of mixing. Such mixing angle plays an important role in the neutrino oscillations in vacuum. The maximum mixing takes place for $\theta_0 = \frac{\pi}{4}$.

If the vacuum mixing angle is 0 or $\frac{\pi}{2}$ there will be no mixing at all. Now let us consider the evolution equation for the vacuum oscillation given by

$$i\partial_t \begin{pmatrix} \nu_e \\ \nu_l \end{pmatrix} = \left[E + \frac{m_1^2 + m_2^2}{4E} + \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta_0 & \sin 2\theta_0 \\ \sin 2\theta_0 & \cos 2\theta_0 \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_l \end{pmatrix}. \quad (2.1)$$

The above equation is obtained in approximating the three flavor case. It has already been stated that the ν_l is taken as the superposition of ν_μ and ν_τ . The approximations are valid because the mixing angle θ_{13} is very small and because two of the mass eigenstates are very close to each other compared to the third one. It is now quite easy to calculate corresponding transition probability. We have taken the initial beam consists of electron neutrinos and therefore the conversion probability becomes

$$P_{el} = |\nu_l|^2 = \sin^2 2\theta_0 \sin^2 \left(\frac{\Delta m^2 t}{4E} \right). \quad (2.2)$$

It is now observed that for $\theta_0 = 0$ or $\frac{\pi}{2}$ there is no flavor transition even if the neutrino has the non-zero mass. In that case each of the flavor eigenstates becomes either of mass eigenstates. It is worth noting that according to equation (2.2) the

flavor transition does not occur too if the neutrino has the zero mass. It is quite clear that the vacuum oscillation depends on neutrino mass as well as mixing angle and therefore, in vacuum the neutrinos can oscillate only when they have the non-zero mass along with the suitable vacuum mixing angle lying between 0 to $\frac{\pi}{2}$.

Let us consider now MSW effect in which both the eigenstates and the eigenvalues, and consequently, the effective mixing angle depend on matter density. In that case the evolution equation is modified. The effective MSW Hamiltonian takes the form

$$H_f = \begin{pmatrix} -\Delta m^2 \cos 2\theta_0 + A & \Delta m^2 \sin 2\theta_0 \\ \Delta m^2 \sin 2\theta_0 & \Delta m^2 \cos 2\theta_0 \end{pmatrix}, \quad (2.3a)$$

where

$$A = 2E\sqrt{2}G_F n_e. \quad (2.3b)$$

The corresponding eigenvalues of this matrix are

$$\tilde{m}_{1,2}^2 = \frac{A}{2} \mp \frac{\sqrt{(\Delta m^2 \cos 2\theta_0 - A)^2 + (\Delta m^2 \sin 2\theta_0)^2}}{2} \quad (2.4a)$$

The effective mass squared difference in presence of matter takes the form

$$\Delta \tilde{m}^2 = \tilde{m}_2^2 - \tilde{m}_1^2 = \frac{\sqrt{(\Delta m^2 \cos 2\theta_0 - A)^2 + (\Delta m^2 \sin 2\theta_0)^2}}{2}. \quad (2.4b)$$

We know that in the framework of the standard model $m_1 = m_2 = 0$ and hence $\Delta m^2 = 0$. But in the matter induced oscillation (2.4a) and (2.4b) imply $\Delta \tilde{m}^2 = A$. It means a non-zero effective mass, proportional to the electron number density, is evolved in presence of matter. Now we shall verify whether such mass is capable of generating any oscillation. The evolution equation for the matter induced oscillation is given by

$$i\partial_t \begin{pmatrix} \nu_e \\ \nu_l \end{pmatrix} = \left[E + \frac{m_1^2 + m_2^2 + A}{4E} + \frac{1}{4E} \begin{pmatrix} -\Delta m^2 \cos 2\theta_0 & \Delta m^2 \sin 2\theta_0 \\ \Delta m^2 \sin 2\theta_0 & \Delta m^2 \cos 2\theta_0 \end{pmatrix} \right. \\ \left. + \frac{A}{4E} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_l \end{pmatrix}. \quad (2.5)$$

In the framework of standard model such equation is reduced to

$$i\partial_t \begin{pmatrix} \nu_e \\ \nu_l \end{pmatrix} = \left[E + \frac{A}{4E} + \frac{A}{4E} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_l \end{pmatrix}. \quad (2.6)$$

We now compare the equation (2.6) with the evolution equation of vacuum oscillation given by (2.1). It seems to us that (2.6) represents the neutrino oscillation in which one of the eigenstate ν_1 is massless and other one ν_2 has an effective mass proportional to the electron number density. It also shows that the corresponding effective mixing angle becomes $\frac{\pi}{2}$ and therefore, the conversion probability is zero, resulting there will be no oscillation at all. Due to such mixing angle the electron flavor coincides with the heavy eigenstate, i.e., $\nu_e \sim \nu_2$; as a result the electron neutrino will gain the effective mass $2E\sqrt{2}G_F n_e$, but fails to mix with ν_l that remains massless as per the standard model consideration. Thus the flavor transition may not be possible even if the electron neutrino gains the non-zero mass. We shall see the situation from other angle.

3. Pendulum in Flavor Space

Let us now consider again the evolution equation of the vacuum oscillation, i.e., the equation (2.1). It can be expressed as

$$i\partial_t \begin{pmatrix} \nu_e \\ \nu_l \end{pmatrix} = \left[E + \frac{M^2}{2E} \right] \begin{pmatrix} \nu_e \\ \nu_l \end{pmatrix}, \quad (3.1)$$

where the mass matrix M^2 is represented by

$$M^2 = \frac{m_1^2 + m_2^2}{2} + \frac{\Delta m^2}{2} \mathbf{B} \cdot \boldsymbol{\sigma}, \quad (3.2)$$

where $\mathbf{B} = (\sin 2\theta_0, 0, -\cos 2\theta_0)$ and $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ with $\sigma_i (i = 1, 2, 3)$ being the Pauli matrices. This $\mathbf{B}$ plays an important role which will be discussed later.

A density matrix is defined by

$$\rho = \begin{pmatrix} \nu_e^* \\ \nu_l^* \end{pmatrix} \begin{pmatrix} \nu_e & \nu_l \end{pmatrix} \quad (3.3)$$

and now it is possible to express the evolution equation of the vacuum oscillation in terms of density matrix in the following form.

$$i\partial_t \rho = [M^2, \rho]. \quad (3.4)$$

The corresponding density matrix can also be represented by

$$\rho = \frac{1}{2}(1 + \mathbf{P} \cdot \boldsymbol{\sigma}). \quad (3.5)$$

This $\mathbf{P}$ plays a crucial role to describe the dynamics of the neutrino oscillation. The z -th component of $\mathbf{P}$ is related to the transition probability as follows.

$$|v_e|^2 = \frac{1}{2}(1 + P_z), \quad (3.5a)$$

$$|v_l|^2 = \frac{1}{2}(1 - P_z). \quad (3.5b)$$

The other two components of $\mathbf{P}$ give the information about the phase factors. $P = 1$ stands for the perfect coherent mixture of neutrino gas, but in reality $P < 1$. Now using the expressions of M^2 and ρ from the equations (3.2) and (3.3), respectively, it can be deduced from the equation (3.4) as

$$\partial_t \mathbf{P} = \omega(\mathbf{B} \times \mathbf{P}), \quad (3.6)$$

where

$$\omega = \frac{\Delta m^2}{2E}. \quad (3.6a)$$

This is analogous to a equation of a spherical pendulum in flavor space in which $\mathbf{P}$ precesses round $\mathbf{B}$ at an angle $2\theta_0$ with a frequency ω . The representation of the vacuum oscillation phenomenon in this form was first considered by Stodolsky [6] and he deduced the equation (3.6). In this case $\mathbf{P}$ stands for the polarization vector which is very much similar to the polarization vector of light. It is now clear that $\mathbf{P} = (0, 0, 1)$ represents the electron flavor and $\mathbf{P} = (0, 0, -1)$ stands for the x flavor. Consequently, $\mathbf{z}$ represents the flavor direction, whereas $\mathbf{B}$ stands as the mass direction. Such $\mathbf{B}$ is also called the external magnetic field as the picture is very much analogous to the spin-precession scenario of the atomic model, of course it has no relation with magnetic field in the real sense. That pendular model was frequently used to discuss the collective oscillation of the supernova neutrinos [7-13].

We now consider the matter induced oscillation. The equation (2.5) shows the evolution equation of the MSW effect. Starting with that equation and following the same procedure as in the case of vacuum oscillation one can reach to the Stodolsky type of equation as

$$\partial_t \mathbf{P} = \omega(\mathbf{B} \times \mathbf{P}) + \lambda(\mathbf{z} \times \mathbf{P}), \quad (3.7)$$

where

$$\lambda = \frac{A}{2E}. \quad (3.7a)$$

It shows the polarization vector does not precess around $\mathbf{B}$ but can be considered to precess around a vector $\mathbf{R} = \omega\mathbf{B} + \lambda\mathbf{z}$ with frequency R . The angle between such vector and $\mathbf{z}$ axis is nothing but the twice of the mixing angle due to the matter induced oscillation. The frequency and the effective mixing angle are calculated as

$$R = \frac{\Delta\tilde{m}^2}{2E} = [(\lambda - \omega \cos 2\theta_0)^2 + \omega^2 \sin^2 2\theta_0]^{1/2} \quad (3.8a)$$

and

$$\cos 2\tilde{\theta} = \frac{\lambda - \omega \cos 2\theta_0}{[(\lambda - \omega \cos 2\theta_0)^2 + \omega^2 \sin^2 2\theta_0]^{1/2}}. \quad (3.8b)$$

If we watch the equation (3.7), we see the second part of that equation seems to remain non-zero even while the vacuum frequency is taken to be zero. Therefore, since the initial choice of $\mathbf{P}$ is arbitrary it seems that the precession can take place even when $\omega = 0$. Does it imply that oscillations occur even if the neutrinos are massless? Eventually, the answer would be negative. $\mathbf{z}$ represents the flavor directions, represented by $(0, 0, 1)$. In case when $\omega = 0$, the equation (3.7) yields $\partial_t P_z = 0$ and thus $P_z = \text{constant} = 1$. Therefore, the survival probability remains 1, which must interpret that there is no oscillation at all.

4. Discussion

From the Stodolsky type of equation it may be wrongly understood that in the framework of standard model, i.e., if one can put $\omega = 0$ by hand the neutrino oscillation is still possible for arbitrary polarization vector. But if we study the Stodolsky type of equation very carefully it states the polarization vector precesses

around the negative mass direction (i.e., $-\mathbf{B}$) in such a way that it must coincide, at a particular stage of its motion, with the flavor direction (here it is z -axis). In case of vacuum oscillation mass direction always makes a constant angle with the flavor direction and therefore, polarization vector precesses with that particular angle. In the matter induced oscillation the angle between effective mass direction and flavor direction is not constant at all, rather it depends on the matter density. Consequently the neutrino oscillation depends not only upon the mass, but it also depends on the mixing angle. In presence of matter both of them are proportional to the electron number density as well as the mass of the neutrinos. But in the framework of standard model when the neutrino is massless in vacuum the presence of matter yields an effective neutrino mass, but fails to generate any oscillation as the effective mixing angle becomes $\frac{\pi}{2}$ resulting no mixing at all. The zero neutrino mass is equivalent to the vanishing vacuum frequency ω in the Stodolsky equation. In this case the presence of matter results a non-zero high frequency λ but the mass direction remains aligned to the flavor direction. For a flavor oscillation, whatever small it may be, it is mandatory that the mass direction gets separated from the flavor direction. As it is not possible for $\omega = 0$, i.e., for zero mass of either of the mass eigenstate no flavor oscillation can take place, even for a high value of effective mass. Thus our analytical study shows that the neutrino oscillation phenomena cannot be explained, even in presence of matter if neutrinos are considered as massless. Although the standard model is very much successful to explain many low as well as high energy phenomena in the particle physics but within the framework of this model it is not possible to realize the neutrino oscillation phenomenon. Presently, there are a number of extended models which can successfully explain the generation of neutrino mass and henceforth the neutrino oscillation.

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