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BIFURCATIONS OF TRAVELLING WAVE SOLUTIONS FOR THE CALOGERO-BOGOYAVLENSKII-SCHIFF EQUATION

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Abstract

By using the bifurcation theory of planar dynamical systems to the Calogero-Bogoyavlenskii-Schiff equation, the existence of solitary wave solution and periodic travelling wave solutions is proved. Exact explicit parametric representations of the above solutions are obtained.

1. Introduction

In this paper, we consider the Calogero-Bogoyavlenskii-Schiff equation of the form

$$u_t + uu_z + \frac{1}{2} u_x \partial_x^{-1} u_z + \frac{1}{4} u_{xxz} = 0, \tag{1}$$

where $\partial_x^{-1} f = \int f dx.$ To remove the integral term in equation (1), we introduce the following potential of the form

$$u(x, z, t) = v_x(x, z, t). \tag{2}$$

Then equation (1) becomes

$$v_{xt} + v_x v_{xz} + \frac{1}{2} v_{xx} v_z + \frac{1}{4} v_{xxxz} = 0. \tag{3}$$

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This system is used to describe the $(2+1)$ dimensional interaction of Riemann wave propagated along the z-axis with long wave propagated along the x-axis [1-4]. The authors [1-2] obtained overturning soliton solutions of equation (1). To the best of our knowledge, the dynamical behavior of travelling wave solutions of (1) has not been performed before. In this paper, by using bifurcation theory of planar dynamical systems, we study exact travelling wave solutions of equation (1) and we show that it has solitary wave solutions and periodic wave solutions. Exact explicit parametric representations of these solutions are given.

Substituting the travelling wave transformation $v(x, z, t) = v(\xi)$, where $\xi = x + z - ct$ with the wave speed c reduces equation (3) to the ordinary differential equation

$$-cv'' + \frac{3}{2}v'v'' + \frac{1}{4}v'''' = 0, \tag{4}$$

where $'$ is the derivative with respect to ξ. Integrating once and neglecting the integral constant, we have

$$-cv' + \frac{3}{4}(v')^2 + \frac{1}{4}v''' = 0. \tag{5}$$

To extend the dynamical method to the system (5), we consider the following transformation

$$v' = \varphi. \tag{6}$$

Substituting equation (6) into equation (5), we obtain

$$-c\varphi + \frac{3}{4}(\varphi)^2 + \frac{1}{4}\varphi'' = 0. \tag{7}$$

We have the following travelling wave system which is a planar dynamic system:

$$\begin{cases} \dfrac{d\varphi}{d\xi} = y, \\ \dfrac{dy}{d\xi} = \varphi(4c - 3\varphi). \end{cases} \tag{8}$$

Obviously, equation (8) is a planar Hamiltonian system with Hamiltonian function

$$H(\varphi, y) = \frac{1}{2}y^2 - 2c\varphi^2 + \varphi^3 = h, \text{ say.} \tag{9}$$

Note that to this physical model, only bounded travelling waves are meaningful, so we just pay our attention to the bounded solutions of the equation (8) which are physically acceptable. To investigate all bifurcations of solitary waves, kink waves and periodic waves of equation (1), we should find all periodic annuli, homoclinic and heteroclinic orbits of the equation (8) depending on the parameter c of the system. The bifurcation theory of dynamical systems (see [5-7]) plays an important role in our study.

The rest of this paper is organized as follows. In Section 2, we consider bifurcations of phase portraits of the planar Hamiltonian system (8). In Section 3, we give some exact explicit parametric representations for travelling wave solutions of equation (1).

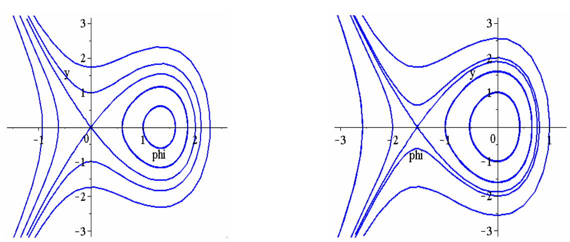
2. Bifurcations of Phase Portraits of Equation (8)

In this section, we will consider phase portraits of the system (8). Let $E_i(\varphi_i, 0)$ be an equilibrium point of the system (8). The system (8) has two equilibrium points $E_0(\varphi_0, 0)$ and $E_1(\varphi_1, 0)$, where $\varphi_0 = 0$ and $\varphi_1 = \frac{4c}{3}$. Let $M(\varphi_i, 0)$ be the coefficient matrix of the linearized system of the system (8) at an equilibrium point $E_i(\varphi_i, 0)$. Then we have

$$J(\varphi_i, 0) = \det M(\varphi_i, 0) = 6\varphi_i - 4c. \tag{10}$$

By the theory of planar dynamical systems (see [5-7]), the equilibrium point $E_i(\varphi_i, 0)$ of the Hamiltonian system (8) is a saddle point if $J(\varphi_i, 0) < 0$; the equilibrium point $E_i(\varphi_i, 0)$ is a center if $J(\varphi_i, 0) > 0$.

From the above analysis, we obtain the different phase portraits of the system (8) shown in Figures 1(a) and 1(b).



(a) (b)

Figure 1. The phase portraits of system (8): (a) $c > 0$, (b) $c < 0$.

3. Exact Explicit Travelling Wave Solutions of Equation (1)

The interest in finding exact solutions [1-4, 8] to nonlinear wave equations by using appropriate technique is growing day by day and these exact solutions play an important role in the study of nonlinear physical phenomena. In this section, by using the travelling wave system (8) and the Hamiltonian (9) with $h = 0$ to do calculations, we obtain the following exact explicit parametric representations of equation (1).

(1) When $c > 0$, (see Figure 1(a)), we have the smooth solitary wave solution of equation (1) of the form

$$u = 2c\,\mathrm{sech}^2(\sqrt{c}\,\xi). \tag{11}$$

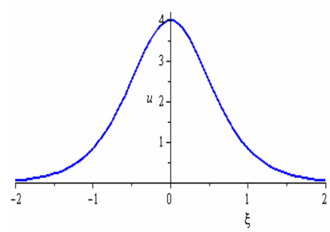


Figure 2. Graph of the solitary wave solution (11) for $c = 2$.

(2) When $c < 0$, (see Figure 1(b)), we have a family of smooth periodic travelling wave solutions of equation (1) of the form

$$u = -2\sqrt{-c}(1 + \tan^2(\sqrt{-c}\,\xi)). \tag{12}$$

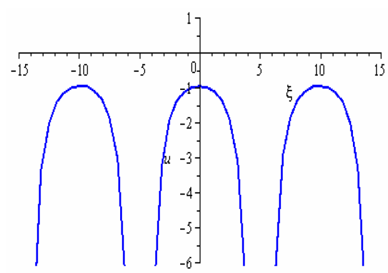


Figure 3. Graph of the periodic wave solution (12) for $c = -0.1$.

4. Conclusion

In this paper, we have considered the bifurcation behavior of travelling wave solutions of equation (1). We have obtained smooth solitary wave solution and smooth periodic travelling wave solutions. The study of bifurcation behavior of the travelling wave solutions of equation (1) will add some value in the literature of the Calogero-Bogoyavlenskii-Schiff equation.

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