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AN ANALYTICAL APPROACH TO THE NON-COMMUTATIVE SPACE EFFECT IN THE BETHE-SALPETER EQUATION FOR TWO PARTICLES' BOUND STATE: QUANTUM ELECTRODYNAMIC MODELLING

ALIREZA HEIDARI[a], SEYEDALI VEDAD[a], O. ANWAR BÉG[b] and MOHAMMADALI GHORBANI[a, *]

[a]Institute for Advanced Studies
Tehran 14456-63543
Iran

[b]Multi-Physical Modelling
Aerospace Engineering Program
Department of Engineering and Mathematics
Sheffield Hallam University
Room 4112, Sheaf Building
Sheffield, S1 1WB, England, UK
e-mail: mohammadalighorbani1983@yahoo.com

Abstract

In this article, the non-commutative space effect on the Bethe-Salpeter equation for two particles' bound state is investigated. In this investigation, the two-particle bound states are considered with spin 0-spin 0, spin $\frac{1}{2}$-spin $\frac{1}{2}$, and spin $\frac{1}{2}$-spin 0 particles. In the energy spectrum, the lowest independent correction from the spin, in all cases starts from the $\theta\alpha^4$ order. At the same time, the spin-dependent correction in the non-commutative space starts from the $\theta\alpha^6$ order.

Keywords and phrases: bound state, energy spectrum, Bethe-Salpeter equation, non-commutative space, spin, quantum electrodynamics modelling.

*Corresponding author

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1. Introduction

The phenomenological aspects of the non-commutative spaces have, recently, been matters of consideration [1-5]. In fact, the most important matter is the way to measure and detecting the non-commutative effect on the physical phenomena. It seems that the quantum electrodynamics in the non-commutative space (NCQED) is the best way to compute such effects. The major difference between QED and NCQED occurs in presence of the new interactions (three and four-photon vortexes) which itself leads to the complexity of the computations. In order to investigate such effects, one can name two-particle bound states like atoms of hydrogen, positronium, etc. Therefore, to do this, in addition to the precise laboratory data, the theoretical computations, too, must be done with great care. Although the Feynman rules are completely known for NCQED [2], in the case of bound states one must use these rules in especial methods, like Bethe-Salpeter or non-relativistic quantum electrodynamics (NRQED) [6].

2. The Bethe-Salpeter Equation

In order to describe a system consisting of a covariant two-particle bound state, one must use the Bethe-Salpeter equation. This equation for the two-particle bound state takes the form

$$\Psi(p, q) = S(p)S(q)\int \frac{d^4k}{(2\pi)^4 i} I(k; p, q)\Psi(p+k, q-k) \tag{1}$$

in which $\Psi(p, q)$ is amplitude for the bound state, $S(p)$ and $S(q)$ are the particle field propagators and $I(k; p, q)$ is the interaction nucleus which, itself, is formed from the combination of all of the non-reducible graphs. One must note that in case of one particle with spin $\frac{1}{2}$, S is the fermionic propagator and for non-spin particles it represents the bosonic propagator. In QED, the ladder approximation has been shown to be a more logical approximation; therefore, in its lowest order, the interaction nucleus can be written as follows:

$$I\Psi = (I_{obe} + I_a)\Psi \tag{2}$$

in which the subscript $(\ )_{obe}$ corresponds to exchanging one boson and $(\ )_a$ corresponds to its annihilation. Of course, I_a exists only in the case of the bound and anti-states of the same particle. Later, in the Feynman gauge, while separating the instantaneous and delayed interactions, the delayed interaction is computed with the aid of the perturbation. To do this, I_{obe} can be written, as an example, for two particles with spin $\frac{1}{2}$ [7] as follows:

$$I_{obe} = -\frac{4\pi\alpha\gamma_1^0\gamma_2^0}{\kappa^2} + 4\pi\alpha\left(\frac{\gamma_1^0\gamma_2^0k_0^2}{k^2\kappa^2} - \frac{\gamma_1 \cdot \gamma_2}{k^2}\right) \tag{3}$$

in which k is the four-momentum and κ is the boson-exchange momentum. In general, I can be written in the form

$$I = V_c + \delta V \tag{4}$$

in which V_c is the interaction nucleus caused by the Coloumb potential and δV is caused by the combination of perturbative potentials. Consequently, equation (1) may be formulated as

$$\Psi(p, q)$$
$$= S(p)S(q)\left[\int \frac{d^4k}{(2\pi)^4 i} V_c\Psi(p+k, q-k) + \int \frac{d^4k}{(2\pi)^4 i}\delta V\Psi(p+k, q-k)\right]. \tag{5}$$

Considering the smallness of the second component, one must, at first, arrive at an answer for equation (6)

$$\Psi(p, q) = S(p)S(q)\int \frac{d^4k}{(2\pi)^4 i} V_c\Psi(p+k, q-k). \tag{6}$$

To do this, one can easily observe that

$$\Psi(p) = D_{\pm\pm}^{-1}\frac{\Lambda_1^{\pm}\Lambda_2^{\pm}}{2i\pi}\upsilon\phi(P) \tag{7}$$

in which we have

$$\upsilon\phi(P) = \frac{\alpha}{2\pi^2}\int \frac{d^3p'}{(P-P')^2}\phi(P'), \tag{8}$$

$$\phi(P) = \int dp^0\Psi(p^0, P) \tag{9}$$

and

$$D_{\pm\pm}^{-1} = \frac{1}{\left[\frac{E}{2} + p_0 \mp (w - i\varepsilon)\right]\left[\frac{E}{2} - p_0 \mp (w - i\varepsilon)\right]}. \tag{10}$$

By integrating from equation (7) on p^0, one can arrive at the wave function $\phi(P)$ which is given by

$$\phi(P)\int dp^0 D_{\pm\pm}^{-1} \frac{\Lambda_1^{\pm}\Lambda_2^{\pm}}{2i\pi} \upsilon\phi(P). \tag{11}$$

Now, as the Coloumb wave function is determined, one can compute the effect of δV perturbation on the energy in the form of commutation in energy as [7]

$$\Delta E = -\frac{1}{(2\pi)^6}\int d^4 p\tilde{\Psi}(p)(\delta V\Psi)(p) \tag{12}$$

in which we have

$$(\delta V\Psi)(p) = \int d^4 p'\delta V(p,\ p')\Psi(p'). \tag{13}$$

If the perturbation does not depend on p and p', or $\delta V(p,\ p') = \delta V(P,\ P')$, then the energy commutation is simplified to

$$\Delta E = -\frac{1}{(2\pi)^6}\int d^3 p\tilde{\phi}(P)(\delta V\phi)(P'). \tag{14}$$

3. The Bethe-Salpeter Equation in Non-commutative Space

Equation (1) is a general equation and its form is independent of the type of interaction. Therefore, to formulate the Bethe-Salpeter equation in NCQED, considering the Feynman rules given in NCQED, we are required to write the interaction nucleus correctly. The difference of QED and NCQED, in addition to the presence of new interactions (three and four-photon), in the phase factors' existence, depends on a momentum which is multiplied in each QED vortex. For instance, the interaction nucleus in the case of two particles with an opposite charge on the tree chart and in the mass center system is

$$I_\theta^l(k;\, p,\, -p) = e^{\frac{i}{2}\cdot P\cdot(\theta_+ + \theta_-)\cdot\kappa} \cdot I^l(k) \tag{15}$$

in which l shows the ladder approximation, and, also, $I^l(k)$ is the interaction nucleus in the non-commutative space which, an example of which (for two particles with spin $\frac{1}{2}$) is given in equation (3) and the non-commutative parameter θ is an anti-symmetric tensor, which is defined as

$$\theta^{\mu\nu} = -i[x^{\mu}, x^{\nu}]. \tag{16}$$

Later, we will assume $\theta^{i} = 0$ since if $\theta^{i} \neq 0$, there will be problems in the uniqueness of the field theories and the concept of causation [8, 9]. Choosing $\theta_{\pm}$ in equation (15) gives the probability of considering different non-commutative parameters for particles with opposite charges.

In general, the interaction nucleus in the non-commutative space, compared to equation (2), can be written as

$$I^{NC}\Psi = (I_{obe}^{NC} + I_{a}^{NC})\Psi \tag{17}$$

in which

$$I_{a}^{NC} = I_{a} \tag{18}$$

and

$$I_{obe}^{NC} = I_{obe} + I'(\theta). \tag{19}$$

One must note that I_a expression exists only when the particle and the anti-particle of the associated particle, are in their bound state. Consequently, in this case, one can easily show that the phase factor which multiplies in each vortex of this graph equals 1. Also, $I'(\theta)$, given in equation (19) shows the interactions caused by the non-commutative space.

3.1. The Bethe-Salpeter equation in the non-commutative space for two particles with spin 0

Here, two particles are with spin 0 and opposite charges form a bound state. Therefore, S in equation (1) is a bosonic propagator and one can arrive at δV with the help of $I'(\theta)$ in the mass center system as

$$I'(\theta) = \frac{4\pi\alpha}{\kappa^{2}}(1 - e^{2ip\wedge p'}) - \frac{4\pi\alpha k_{0}^{2}}{k^{2}\kappa^{2}}(1 - e^{2ip\wedge p'}) \tag{20}$$

in which $p' = p - k$. One must note that the lowest energy change by the potential given in equation (20) is of the order, $\theta\alpha^4$. Actually, each momentum factor in the potential leads to adding one α to the energy. Now, one can compute the change in energy using equation (12) in its lowest order as

$$\Delta E = -\frac{1}{(2\pi)^6}\int d^3p\int d^3p'\phi^*(P)\left[\frac{4\pi\alpha}{(P-P')^2}(1-e^{2ip\wedge p'}\right]\phi(P'). \tag{21}$$

Using the wave function Fourier transformation, one can change the above expression, via some algebraic operations to

$$\Delta E = -4\pi\alpha\int d^3r\phi^*(r)\left[\frac{1}{4\pi r}\phi(r) - \phi(r + i\theta\cdot\nabla)\frac{1}{4\pi r}\right]. \tag{22}$$

Up to the first-order of θ, we will have

$$\Delta E = \alpha\int d^3r\phi^*(r)\left[-\frac{r\cdot\theta\cdot i\nabla}{r^3}\phi(r)\right]. \tag{23}$$

Also, for two optional vectors A and B, we have

$$A\cdot\theta\cdot B = \Theta\cdot(A\times B) \tag{24}$$

in which

$$\Theta = (\theta^{23}, \theta^{31}, \theta^{12}). \tag{25}$$

Consequently, a change in the energy up to the lowest order of θ is given by

$$\Delta E = \alpha\int d^3r\phi^*(r)\left(\frac{\Theta\cdot L}{r^3}\right)\theta(r). \tag{26}$$

By choosing Θ in the direction of z, we arrive at

$$\Delta E = \alpha^4\Theta\frac{P_{n,l}}{l\left(l+\frac{1}{2}\right)(l+1)} \tag{27}$$

in which $P_{n,l}$ is a polynomial from the quantum numbers n and l.

3.2. The Bethe-Salpeter equation in the non-commutative space for two particles with spin $\frac{1}{2}$

In this section, the bound state of two particles with spin $\frac{1}{2}$, e.g., positronium, is investigated. Consequently, S in equation (1) is a fermionic propagator and δV, here, can be reached by determining $I'(\theta)$ with the aid of equation (3) as

$$I'(\theta) = \frac{4\pi\alpha\gamma_1^0\gamma_2^0}{\kappa^2}(1 - e^{2ip\wedge p'}) - 4\pi\alpha\left(\frac{\gamma_1^0\gamma_2^0 k_0^2}{k^2\kappa^2} - \frac{\gamma_1 \cdot \gamma_2}{k^2}\right)(1 - e^{2ip\wedge p'}). \quad (28)$$

Here, a similar trend arises as encountered earlier; the first sentence is of order $\theta\alpha^4$ and can be easily computed. The difference between the present collection with the non-spin state lies in the spin-dependent sentences present in the interaction nucleus (third sentence). These sentences are in the lowest order of $\theta\alpha^6$. To compute the change in energy, one must use equation (12)

$$\Delta E = -\frac{1}{(2\pi)^6}\int d^4p\int d^4p'\psi^*(p)$$

$$\times\left[-4\pi\alpha\left(\frac{\gamma_1^0\gamma_2^0 k_0^2}{k^2\kappa^2} - \frac{\gamma_1 \cdot \gamma_2}{k^2}\right)(1 - e^{2ip\wedge p'})\right]\psi(p'). \quad (29)$$

Inspection of the above equation, reveals that the computation of change determination in energy is much more complex due to the dependence of the interaction nucleus on p and p'. Owing to the fact that in this article the computation of the lowest change in energy is a priority, for the sake of facilitating computations, calculations are performed in the Coloumb gauge. In this gauge, the photonic propagator is defined by

$$\begin{cases} D_{\mu\nu} = \dfrac{-ig_{\mu\nu}}{k^2} \to D_{00} = \dfrac{-i}{\kappa^2}, \\ D_{ij} = \dfrac{-i}{k^2}\left(\delta_{ij} - \dfrac{k_ik_j}{\kappa^2}\right), D_{i0} = D_{0j} = 0. \end{cases} \quad (30)$$

The second sentence in equation (30) can be written as

$$D_{ij} = \frac{i}{\kappa^2}\left(\delta_{ij} - \frac{k_ik_j}{\kappa^2}\right) + \frac{-ik_0^2}{k^2\kappa^2}\left(\delta_{ij} - \frac{k_ik_j}{\kappa^2}\right) \quad (31)$$

in which the first sentence is related to the soft photons propagator and the second sentence, the order of which is smaller than the first one, describes the super soft photons propagator. Therefore, a change in the energy in the lowest order of θ is computed through the following equation

$$\Delta E = -\frac{4\pi\alpha}{(2\pi)^6}\int d^4p'\int d^4p\tilde{\Psi}(p')$$

$$\times\left[\frac{-\gamma_1^0\gamma_2^0}{\kappa^2}+\frac{\gamma_1^i\gamma_2^j}{\kappa^2}\left(\delta_{ij}-\frac{k_ik_j}{\kappa^2}\right)\right](1-e^{2ip\wedge p'})\Psi(P). \tag{32}$$

Considering the following equation

$$\Lambda_1^+\Lambda_2^+\begin{pmatrix}\chi_1\\0\end{pmatrix}\otimes\begin{pmatrix}\chi_2\\0\end{pmatrix}\propto\begin{pmatrix}\chi_1\\ \frac{\sigma_1\cdot p}{E+m_1}\chi_1\end{pmatrix}\otimes\begin{pmatrix}\chi_2\\ \frac{\sigma_2\cdot p}{E+m_2}\chi_2\end{pmatrix}=u_1(p)\otimes u_2(p), \tag{33}$$

ΔE can be written as

$$\Delta E=\frac{-4\pi\alpha}{(2\pi)^6}\int d^3p'd^3p\psi^*(P')\bar{u}_1(P')\bar{u}_2(P')\left[\frac{-\gamma_1^0\gamma_2^0}{\kappa^2}+\frac{\gamma_1^i\gamma_2^j}{\kappa^2}\left(\delta_{ij}-\frac{k_ik_j}{\kappa^2}\right)\right]$$

$$\times(1-e^{2ip\wedge p'})u_1(P)u_2(P)\psi(P) \tag{34}$$

in which

$$\psi(P)=\int dp^0D_{++}^{-1}\frac{1}{2i\pi}\upsilon\phi(P). \tag{35}$$

To reach equation (34), one must consider the fact that $\Lambda_1^+\Lambda_2^-=\Lambda_1^-\Lambda_2^+=0$ and $\Lambda_1^+\Lambda_2^+$ are much bigger than $\Lambda_1^-\Lambda_2^-$. Now, using the following equations, one can compute the change in energy

$$\bar{u}(P')\gamma_iu(P)=-\chi'^{\dagger}\frac{(P+P')}{2m}\chi+i\chi'^{\dagger}\frac{(P'-P)\times\sigma}{2m}\chi+O\left(\frac{P^2}{m^2}\right), \tag{36}$$

also

$$\bar{u}(P')\gamma_0u(P)=\chi'^{\dagger}\chi+i\chi'^{\dagger}\frac{(P'\times P)\cdot\sigma}{4m^2}\chi-\chi'^{\dagger}\frac{(P-P')^2}{8m^2}\chi+O\left(\frac{P^3}{m^3}\right) \tag{37}$$

in which the two-component spinor χ is normalized to 1. Consequently, now, for ΔE, we have

$$\Delta E=4\pi\alpha\int\frac{d^3p'd^3p}{(2\pi)^6}\psi^*(P')\sum_i\Gamma_i(P,\ P')(1-e^{2ip\wedge p'})\psi(P) \tag{38}$$

and

$$\Gamma_1 = \frac{1}{\kappa^2}, \tag{39}$$

$$\Gamma_2 = i\frac{(P' \times P)\cdot\sigma_1}{4m_1^2}\frac{1}{\kappa^2}, \tag{40}$$

$$\Gamma_3 = \Gamma_2(1 \to 2), \tag{41}$$

$$\Gamma_4 = -\frac{(P - P')^2}{8m_1^2}\frac{1}{\kappa^2}, \tag{42}$$

$$\Gamma_5 = \Gamma_4(1 \to 2), \tag{43}$$

$$\Gamma_6 = \frac{(P + P')_i}{2m_1}\left[\frac{-1}{\kappa^2}\left(\delta_{ij} - \frac{k_i k_j}{\kappa^2}\right)\right]\frac{(P + P')_j}{2m_2}, \tag{44}$$

$$\Gamma_7 = \frac{(P + P')_i}{2m_1}\left[\frac{1}{\kappa^2}\left(\delta_{ij} - \frac{k_i k_j}{\kappa^2}\right)\right]\left[\frac{i(P' - P)\times\sigma_1}{2m_2}\right]_j, \tag{45}$$

$$\Gamma_8 = \left[\frac{i(P' - P)\times\sigma_2}{2m_2}\right]_i\left[\frac{1}{\kappa^2}\left(\delta_{ij} - \frac{k_i k_j}{\kappa^2}\right)\right]\frac{(P + P')_j}{2m_1}, \tag{46}$$

$$\Gamma_9 = \left[\frac{i(P' - P)\times\sigma_2}{2m_2}\right]_i\left[\frac{-1}{\kappa^2}\left(\delta_{ij} - \frac{k_i k_j}{\kappa^2}\right)\right]\left[\frac{i(P' - P)\times\sigma_1}{2m_1}\right]_j. \tag{47}$$

In equation (38), the sentence Γ_1 starts from order $\theta\alpha^4$, and its value is computed in equations (26) and (27). The whole other sentences' orders $(\Gamma_2 - \Gamma_9)$ start from $\theta\alpha^6$. Therefore, to determine the greatest change in energy, one must just consider the first sentence. However, to determine the non-commutative effect on some important physical phenomena which are measured with high precision in laboratories, such as the ultra-small slit in a positronium atom $E(2^3 S_1) - E(2^3 p_2)$ [10-13], the first sentence does not exert any tangible influence. In this case, the spin-dependent sentences $\Gamma_2, \Gamma_3, \Gamma_7, \Gamma_8,$ and Γ_9 change the value of the ultra-small slit compared to ordinary space. Consequently, by computing the change in energy for the above-mentioned sentences, one can easily compute the ultra-small slit positronium $(m_1 = m_2 = m_e)$ in the non-commutative space

$$\Delta E_2 = -\frac{i\pi}{m_e^2}\int\frac{d^3p'd^3p}{(2\pi)^6}\psi^*(P')\frac{P \times P' \cdot \sigma_1}{\kappa^2}(1 - e^{2ip\wedge p'})\psi(P). \tag{48}$$

Following several algebraic operations, we arrive at

$$\Delta E_2 = -\frac{\alpha}{4m_e^2}\int d^3 r\left[\psi^*(r) - \psi^*(r + i\theta \cdot \nabla)\frac{r \times P \cdot \sigma_1}{r^3}\right]\psi(r), \tag{49}$$

$$\Delta E_2 = -\frac{3\alpha}{2m_e^2}\int d^3 r[\Theta \cdot L\psi^*(r)]\frac{S_1 \cdot L}{r^5}\psi(r). \tag{50}$$

For the rest of the sentences one can show that

$$\Delta E_3 = \Delta E_2(S_1 \to S_2) \tag{51}$$

and

$$\Delta E_2 + \Delta E_3 = \frac{1}{2}(\Delta E_7 + \Delta E_8) = -\frac{3\alpha}{2m_e^2}\int d^3 r[\Theta \cdot L\psi^*(r)]\frac{S \cdot L}{r^5}\psi(r) \tag{52}$$

in which $S = S_1 + S_2$. Therefore, equation (52) for the case where $S = 1$, has a non-zero value. The mathematical expression equal with sentence ΔE_9 in both total spin states is zero. Now, by the help of the computed energies, the ultra-small slit positronium atom in the non-commutative space is

$$\Delta E(S = 1) - \Delta E(S = 0) = -\frac{9\alpha}{2m_e^2}\int d^3 r\left[\frac{\Theta \cdot L}{r^5}\psi^*(r)\right]l\psi(r). \tag{53}$$

3.3. The Bethe-Salpeter equation in the non-commutative space for two particles with spin 0 and spin $\frac{1}{2}$

In this section, the case of a two-particle bound state with spins 0 and $\frac{1}{2}$ is investigated. Consequently, in this state, one of the propagators is fermionic and the other is bosonic. Owing to the fact that in this state in one vortex, we have γ_μ and in the other we have one momentum factor, the change in energy is given by

$$E = -\frac{4\pi\alpha}{(2\pi)^6}\int d^4 p'\int d^4 p\tilde{\Psi}(p')$$

$$\times\left[\frac{-\gamma_1^0 p_2^0}{\kappa^2} + \frac{\gamma_1^i p_2^j}{\kappa^2}\left(\delta_{ij} - \frac{k_i k_j}{\kappa^2}\right)\right](1 - e^{2ip \wedge p'})\psi(P) \tag{54}$$

in which

$$p_2 = (p_0 + p_0', P + P'). \tag{55}$$

To achieve a correct normalization in a non-relativistic limit, one must divide the present four-momentum on the scalar vortex to $\sqrt{2E}\sqrt{3E'}$. Therefore, the scalar vortex $\frac{1}{m}$ extension will be

$$\frac{p_0 + p_0'}{\sqrt{2E}\sqrt{2E'}} = 1 + \mathrm{O}\left(\frac{p^4}{m^4}\right), \tag{56}$$

$$\frac{P + P'}{\sqrt{2E}\sqrt{2E'}} = \frac{P + P'}{2m} + \mathrm{O}\left(\frac{p^3}{m^3}\right). \tag{57}$$

Consequently, ΔE for two particles with spins 0 and $\frac{1}{2}$ is written the same as equation (38)

$$E = 4\pi\alpha \int \frac{d^3 p' d^3 p}{(2\pi)^6} \psi^*(P') \sum_i \Gamma_i (P, P') (1 - e^{2ip \wedge p'}) \psi(P) \tag{58}$$

in which Γ_i 's are defined as

$$\Gamma_1 = \frac{1}{\kappa^2}, \tag{59}$$

$$\Gamma_2 = i \frac{(P' \times P) \cdot \sigma_1}{4m_1^2} \frac{1}{\kappa^2}, \tag{60}$$

$$\Gamma_3 = -\frac{(P - P')^2}{8m_1^2} \frac{1}{\kappa^2}, \tag{61}$$

$$\Gamma_4 = \frac{(P + P')_i}{2m_1} \left[\frac{1}{\kappa^2}\left(\delta_{ij} - \frac{k_i k_j}{\kappa^2}\right)\right] \frac{(P + P')_j}{2m_2}, \tag{62}$$

$$\Gamma_5 = \left[\frac{i(P' - P) \times \sigma_1}{2m_1}\right]_i \left[-\frac{1}{\kappa^2}\left(\delta_{ij} - \frac{k_i k_j}{\kappa^2}\right)\right] \frac{(P + P')_j}{2m_2}. \tag{63}$$

Here, too, the first sentence, Γ_1, the value of which is determined by equation (26), starts from order, $\theta\alpha^4$ and is the dominant effect in determining the value of ΔE. The rest of the sentences start from order $\theta\alpha^6$ and are much smaller than the first sentence. Therefore, just computing the spin-dependent sentences Γ_2 and Γ_5 suffices and this leads to

$$\Delta E_2 = -\frac{3\alpha}{2m_1}\int d^3r[\Theta \cdot L\psi^*(r)]\frac{S_1 \cdot L}{r^5}\psi(r) \tag{64}$$

and

$$\Delta E_5 = -\frac{3\alpha}{m_1 m_2}\int d^3r[\Theta \cdot L\psi^*(r)]\frac{S_1 \cdot L}{r^5}\psi(r). \tag{65}$$

4. Conclusion

In this article, two particles' bound state with spins 0 and $\frac{1}{2}$ has been investigated by the help of the Bethe-Salpeter equation. The state of spin 0-spin 0 and the state of spin $\frac{1}{2}$-spin $\frac{1}{2}$ have previously been investigated using different methods. In addition, the results achieved in this article, offered in equation (26) and (53), are fully compatible with the previous methods. Owing to the fact that equivalence of the NRQED method and the Bethe-Salpeter equation have not been shown so far, the full compatibility of the answers for the problem solved in this article can, by itself, possess importance. In this article, also, for the first time, the spin 0-spin $\frac{1}{2}$ state has been investigated using the Bethe-Salpeter equation, the results of which are given in equations (64) and (65). In general, the importance of the Bethe-Salpeter method lies in this fact that the two-particle bound states in the non-commutative space, in general state (different masses and spins) can be investigated systematically.

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