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A NOTE ON MULTIVARIATE EXTENSIONS OF CERTAIN BIVARIATE COPULAS

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In this paper, we consider natural multivariate extensions of originally two-dimensional singular mixture copulas as discussed recently in [1] and [2].

1. Example

We start with [2], Case 2. Assume that U and V are independent uniformly distributed random variables over $[0, 1]$. For $i = 1, \ldots, n$ with $n \in \mathbb{N}$ define

$$T_i := U^{\alpha_i} \cdot V^{\beta_i} \quad \text{with} \quad \alpha_i, \beta_i > 0, \quad \alpha_i \neq \beta_i, \quad i = 1, \ldots, n.$$

Then, according to [2], p. 182

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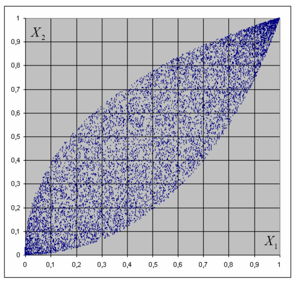
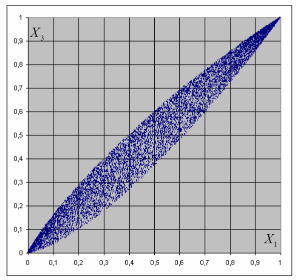
$$X_i := \frac{\alpha_i}{\alpha_i - \beta_i} T_i^{1/\alpha_i} - \frac{\beta_i}{\alpha_i - \beta_i} T_i^{1/\beta_i}, \quad i = 1, \ldots, n$$

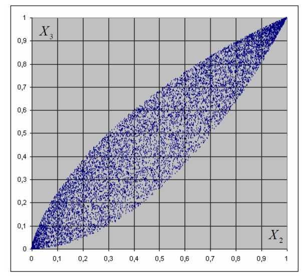
are also uniformly distributed random variables over $[0, 1]$, i.e., $\mathbf{X} := (X_1, \ldots, X_n)$ is a representative of an n-dimensional copula. The following graphs show empirical copulas from this construction for $n = 3$:



$\alpha_1 = 2,\ \beta_1 = 1.1$
$\alpha_2 = 1,\ \beta_2 = 2$

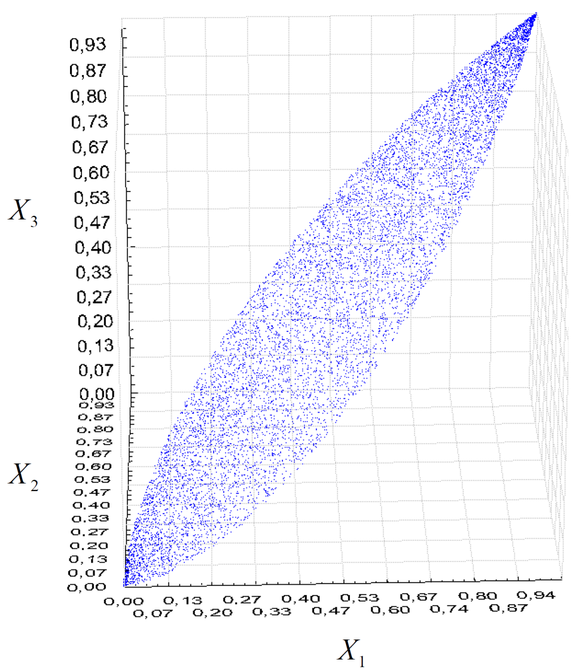


$\alpha_2 = 1,\ \beta_2 = 2$
$\alpha_3 = 1.75,\ \beta_3 = 1.5$



$\alpha_2 = 1,\ \beta_2 = 2$
$\alpha_3 = 1.75,\ \beta_3 = 1.5$

The following graph shows a 3D plot of the empirical copula:



2. Example

Here we consider singular mixture copulas as in [1], Example 1. For this purpose, assume that X is a uniformly distributed random variable over $[0, 1]$. Define

$$Y := F^{-1}(X;\, a_1,\, b_1)$$

with

$$a_1 := c_1 + c_1 \cdot (1 - c_1) \cdot \gamma_1, \quad b_1 := c_1 + c_1 \cdot (1 - c_1) \cdot \delta_1, \quad \text{(cf. [1], p. 126)}$$

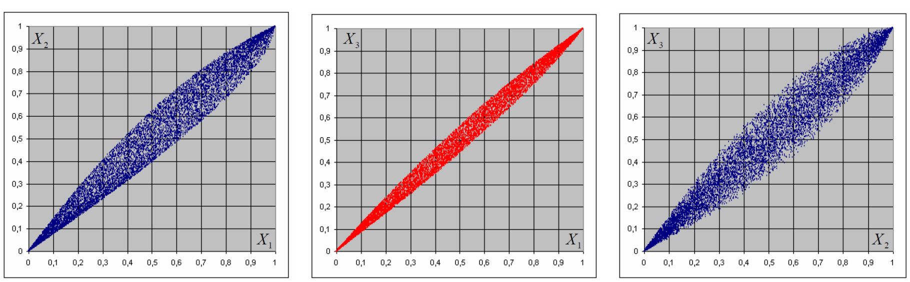
where c_1 is a uniformly distributed random variable over $[0, 1]$, independent of X, and $\gamma_1 := 0.5$, $\delta_1 := 1$. Define further

$$Z := F^{-1}(X;\, a_2,\, b_2)$$

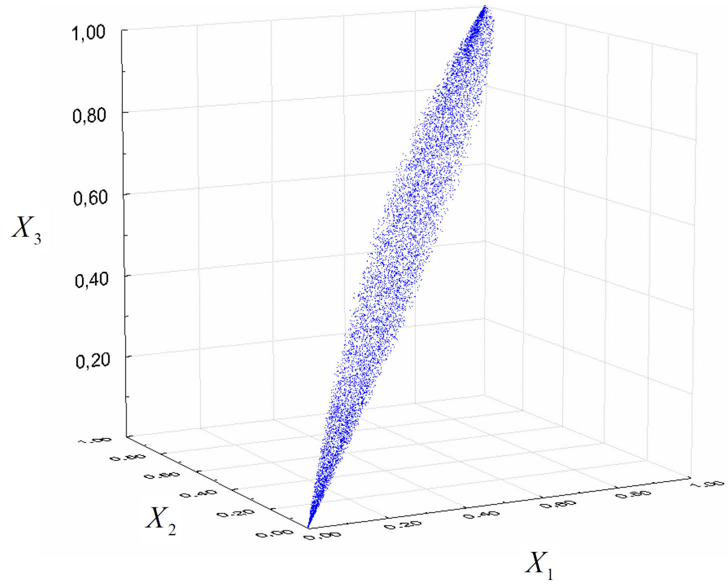
with

$$a_2 := c_2 + c_2 \cdot (1 - c_2) \cdot \gamma_2, \quad b_2 := c_2 + c_2 \cdot (1 - c_2) \cdot \delta_2,$$

where c_2 is another uniformly distributed random variable over $[0, 1]$, independent of X and c_1, and $\gamma_2 := 0.25$, $\delta_2 := 0.5$. Then $\mathbf{X} := (X_1, X_2, X_3)$ represents a 3-dimensional copula. The following graphs show empirical copulas from this construction:



The following graph shows a 3D plot of the empirical copula:



A possible extension to higher dimensions is obvious.

References

[1] D. Pfeifer, Some reflections on singular mixture copulas, Fundamental J. Math. Math. Sci. 19(1) (2025), 123-130.

[2] D. Pfeifer, Tail-dependence properties of some new types of copula models (part II), Fundamental J. Math. Math. Sci. 19(2) (2025), 177-187.