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THE QUESTION OF $E = mc^2$ AND UNIFICATION OF ELECTROMAGNETISM AND GRAVITATION

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Abstract

Einstein's relativity is analyzed and further promoted. Since the electromagnetic energy is not equivalent to mass, validity of $E = mc^2$ is only conditional. To satisfy $E = mc^2$, the photons must have a gravitational wave component. For the dynamic case of massive sources, explicit calculations illustrate that the Einstein equation has no bounded dynamic solutions. Thus, the positive mass theorem of Yau and Schoen is misleading in physics. Moreover, for the dynamic case the linearized equation is incompatible with the Einstein equation, but compatible with Lorentz-Levi-Einstein equation that added a gravitational energy-stress tensor with an anti-gravity coupling to the source. Thus, the assumption of the unique coupling sign for the space-time singularity theorems of Hawking and Penrose is invalid. General relativity is also incomplete because of the absence of the radiation reaction force. When charged particles are involved, there is the charge-mass interaction that necessarily implies Einstein's conjecture of unification between gravitation and electromagnetism; and would explain the weight reduction of charged capacitors. Due to misinterpreting $E = mc^2$ as unconditional, the charge-mass force was overlooked. This is a popular error and even Nobel Laureates 't Hooft and Wilczek made such errors in their Nobel speeches.

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Moreover, since the mass-charge interaction has been neglected in microscopic problems, Einstein's view that quantum mechanics is not a final theory now have additional supports.

"Unthinking respect for authority is the greatest enemy of truth." – A. Einstein

1. Introduction

In 1905, after Special Relativity, Einstein [1] shows, "If a body gives off the energy L, in the form of radiation, its mass diminishes L / c^2". This fact is expressed by the formula $E = mc^2$, where m is the mass of a body and E is the total energy of the body. This formula becomes famous because the fusion and fission in nuclear physics support $\Delta E = \Delta mc^2$, where Δm is the mass difference and ΔE is the energy difference.

Furthermore, Einstein interpreted this formula derived for a special case extended as a general case between mass and energy. He [2] claimed, "We can reverse the relation and say that an increase of E in the amount of energy must be accompanied by an increase of E / c^2 in the mass." However, Einstein's extension actually has no verification. In fact, Einstein [3] has tried very hard for years to extend the formula for an arbitrary E, but failed.

It turns out that Einstein's speculative conjecture is incorrect and has become a theoretical obstacle for the progress in physics. We shall show the details in this paper.

2. Unconditional $E = mc^2$ and Disagrees with General Relativity

In general relativity, the Einstein field equation is,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -KT_{\mu\nu}, \tag{1}$$

where the energy-stress tensor $T_{\mu\nu}$ is the sum of any type of energy-stress tensor. The electromagnetic energy-stress tensor, being traceless, cannot affect R in Equation (1). Therefore, the electromagnetic energy is not equivalent to mass.

One may ask, did not that Einstein prove that electromagnetic energy is equivalent to mass in his 1905 paper [1]. The answer is no. What Einstein has proved is that the photons considered as massless particles can be equivalent to mass. Thus, Einstein's original assumption that photons have only electromagnetic energy must be modified if $E = mc^2$ is applicable for photons. In other words, the photons must also consist of some other non-electromagnetic energy.

When Einstein proposed the notion of photon, it consisted of a quantum of electromagnetic energy, and it acts like a particle in the photoelectric effect. Moreover, it has been observed that the particle π^0 meson decays into two photons (i.e., $\pi^0 \rightarrow \gamma + \gamma$). This was mistakenly considered as evidence that the electromagnetic energy is equivalent to mass. However, there would be a conflict if a photon includes only electromagnetic energy since the electromagnetic energy-stress tensor is traceless. Therefore, the photons must consist of also non-electromagnetic energy.

Since a charged particle is massive, it is natural to consider that the photons consist of electromagnetic energy and gravitational energy. Explicit calculation [4] shows that the photon energy-stress tensor $T(L)_{\mu\nu}$ is, indeed, the sum of the energy stress of the electromagnetic wave $T(E)_{\mu\nu}$ and that of a gravitational wave component $T(g)_{\mu\nu}$, i.e.,

$$T(L)_{\mu\nu} = T(E)_{\mu\nu} + T(g)_{\mu\nu}, \text{ or } T(g)_{\mu\nu} = T(E)_{\mu\nu} - T(L)_{\mu\nu}. \tag{2}$$

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -K[T(E)_{\mu\nu} - T(L)_{\mu\nu}]. \tag{3}$$

Note that if the term $T(L)_{\mu\nu}$ was absent from Equation (3), it would not be possible to have a gravitational wave solution for Equation (3). Note also that the coupling of is an anti-gravity coupling. In fact, the existence of an anti-gravity coupling is a general feature when the gravitational wave is present (see Section 3).

Thus, clearly gravity is no longer just a macroscopic phenomena as Hawking and Penrose claimed [5], but also a microscopic phenomena of crucial importance. Note that both quantum theory and relativity are based on the phenomena of light. The gravity of photons finally shows that there is a link between them. It is gravity that makes the notion of photons compatible with electromagnetic waves [4]. Einstein probably would smile heartily since his theories confirm the link that relates gravity to quantum theory.

3. Misinterpretation of $E = mc^2$ as Unconditional and Errors in General Relativity

If $E = mc^2$ was unconditionally valid, then all the coupling constant in the Einstein equation should have the same sign. It thus follows that according to the space-time singularity theorems of Hawking and Penrose, there must be singularities. However, if $E = mc^2$ is only conditionally valid, the coupling signs may be different. Then, their singularity theorems are irrelevant to physics since it is based on an invalid assumption in physics.

To this end, it is crucial to show whether the Einstein equation has a dynamic solution as Einstein claimed. On this issue, Einstein and Gullstrand [5], the Chairman (1922-1929) of the Nobel Committee for Physics are in opposition to each other. To clarify the issue, one must be clear on some obscure errors.

It was believed that the linearized Einstein equation with massive sources would give a first order approximation of the solution for the Einstein Equation [6] of 1915. The Einstein equation is

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -KT(m)_{\mu\nu} \tag{4}$$

where $T(m)_{\mu\nu}$ is the energy-stress tensor for massive matter, and K is the coupling constant.

The linearized Einstein equation [6] with the linearized harmonic gauge $\partial^{\mu}\bar{\gamma}_{\mu\nu} = 0$ is

$$\frac{1}{2}\partial^{\alpha}\partial_{\alpha}\bar{\gamma}_{\mu\nu} = \kappa T(m)_{\mu\nu}, \quad \text{where } \bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\gamma \text{ and } \gamma = \eta^{\alpha\beta}\gamma_{\alpha\beta}. \tag{5}$$

Note that we have

$$G_{\mu\nu} = G_{\mu\nu}{}^{(1)} + G_{\mu\nu}{}^{(2)} \text{ and } G^{(1)}_{\mu\nu} = \frac{1}{2}\partial^{\alpha}\partial_{\alpha}\bar{\gamma}_{\mu\nu} - \partial^{\alpha}\partial_{\mu}\bar{\gamma}_{\nu\alpha} - \partial^{\alpha}\partial_{\nu}\bar{\gamma}_{\mu\alpha} + \frac{1}{2}\eta_{\mu\nu}\partial^{\alpha}\partial^{\beta}\bar{\gamma}_{\alpha\beta}, \tag{6}$$

where $G_{\mu\nu}{}^{(2)}$ represents the second order terms. This linearized equation is obtained by neglecting the second order terms in the non-linear Einstein equation with a harmonic gauge. Thus, it is expected that its solution is the first approximation of a solution of the Einstein equation. Moreover, this result has been verified for the static case [6].

Many believed the linearized equation would give the first order approximation for all circumstances. However, such a conjecture has never been verified. Moreover, it has been proven not true for the dynamic case [7]. In fact, when gravitational waves are involved, there is no bounded dynamic solution if all the couplings have the same sign [7-9]. For a linear equation such as the Maxwell equation, a weak solution always exists if the sources are weak. However, for a non-linear equation, there is no compelling reason that a bounded weak solution exists for a weak source.

Since a linearized equation such as Equation (4) always produces a bounded solution, if the non-linear Einstein equation has only unbounded solutions, the Einstein equation and its linearized equation would have no compatible solutions. Thus, if a bounded solution does not exist, then the procedure of "linearization" is not valid. Then, in contrast to common naive belief, the non-linear Einstein equation and its "linearization" are essentially independent equations.

A current mistake is the belief that the solution of the linearized equation is mathematically an approximation of the non-linear equation. However, for the dynamic case, this is not satisfied by the Einstein Equation [7-9] because the non-linear equation has no bounded dynamic solution.

A crucial discovery is that for the existence of dynamic solutions there must be different signs for the couplings [7]. For the dynamic case, a modified Einstein equation [7-9] is,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -K[T(m)_{\mu\nu} - t(g)_{\mu\nu}], \tag{7}$$

where $t(g)_{\mu\nu}$ is the energy-stress tensors for gravity. This is the Lorentz-Levi-Einstein equation proposed by Lorentz [10] in 1916 and Levi-Civita [11] in 1917. However, Einstein [12] rejected their proposal on the ground that his equation implies $t(g)_{\mu\nu} = 0$. Now, Einstein is proven clearly wrong.

Note that $t(g)_{\mu\nu}$ with the anti-gravity coupling has appeared already in Equation (2) [4]. In conclusion, the space-time singularity theorems of Hawking and

Penrose are irrelevant to physics.

3. An Example for the Non-existence of a Dynamic Solution

Since the existence of a dynamic solution for the Einstein equation was claimed by Einstein, due to inadequacy in mathematics, many theorists also made this error and refused to accept that Einstein can be wrong [13-17]. Wald [14] and the Wheeler School [13] are the better known among them. This should be expected since Einstein's confidence on his theory was based on his remarkable calculation on the remaining perihelion of Mercury. However, although the calculation suggested that Einstein was close to the truth, as Gullstrand [5] pointed out that he was still not there because his calculation cannot be derived from a many-body problem.

Also, according to the positive mass theorems of Yau and Schoen [18, 19], which is also based on the assumption of unique coupling sign, many absurdly claimed that Einstein's theory is self-consistent and stable (see Section 6).

Here, before pointing out what went wrong in those claims, to prepare a better position to accept the fact that Einstein was wrong on this issue, let us first consider an explicit example that illustrated the non-existence of a dynamic solution. For instance, the metric obtained by Bondi et al. [20] is as follows:

$$ds^2 = e^{2\varphi}\left(d\tau^2 - d\xi^2\right) - u^2\begin{bmatrix}\cosh 2\beta\left(d\eta^2 + d\zeta^2\right) \\ + \sinh 2\beta \cos 2\theta\left(d\eta^2 - d\zeta^2\right) \\ - 2\sinh 2\beta \sin 2\theta d\eta d\zeta\end{bmatrix}, \tag{8a}$$

where ϕ, β and θ are functions of $u(= \tau - \xi)$. It satisfies the differential equation (i.e., their Equation [2.8]),

$$2\phi' = u\left(\beta'^2 + \theta'^2 \sinh^2 2\beta\right), \tag{8b}$$

a special case of $G_{\mu\nu} = 0$. They claimed this is a wave from a distant source. Note that the function ϕ cannot be a periodic function. The metric is irreducibly unbounded because of the factor u^2. Thus, for this case, there is no bounded dynamic solution.

Moreover, when gravity is absent, it needs to have $2\phi = \sinh 2\beta = \sin\ 2\theta = 0$.

These reduce (8a) to

$$ds^2 = \left(d\tau^2 - d\xi^2\right) - u^2\left(d\eta^2 - d\zeta^2\right). \tag{8c}$$

And this metric cannot be reduced to the flat metric. Thus, metric (8c) violates the principle of causality. Moreover, unlike the Schwarzschild solution for the static case, in (8a) there is no physical parameter to be adjusted such that metric (8a) becomes equivalent to the flat metric.

Metric (8) is not a bounded dynamic solution, and this illustrates that the non-linear Einstein equation and the linearized equation are independent equations. Moreover, linearization of (8b) does not make sense since variable *u* is not bounded. Thus, many theorists claim Einstein's notion of weak gravity invalid.

However, Bondi et al. [20] failed to see that there is no bounded dynamic solution. Instead, they challenge the view that both Einstein's notion of weak gravity and his covariance principle are valid. Thus, the editors of the "Royal Society Proceedings A" and the "Physical Review D" support different views since the latter view is supported by Physical Review D. The Royal Society correctly pointed out [21-23] that Einstein's notion of weak gravity is inconsistent with his covariance principle.

5. Errors of Wald and the Wheeler School on the Issue of Dynamic Solutions

According to Einstein [6], in general relativity weak sources would produce a weak field, i.e.,

$$g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}, \text{ where } 1 >> \gamma_{\mu\nu} \tag{9}$$

and $\eta_{\mu\nu}$ is the flat metric when there is no source. However, this is true only if the equation is valid in physics. Many failed to see that a physical requirement may not be satisfied by a mathematical field Equation [24]. When the Einstein equation has a weak solution, an approximate weak solution can be derived through the approach of the field equation being linearized. However, the non-linear equation may not have a bounded solution. The linearized Einstein equation with the linearized harmonic gauge $\partial^{\mu}\bar{\gamma}_{\nu\alpha} = 0$ is Equation (4). Then, the linearized vacuum Einstein equation means

$$G^{(1)}_{\mu\nu}[\gamma^{(1)}_{\alpha\beta}] = 0. \tag{10}$$

Thus, as pointed out by Wald [14], in order to maintain a solution of the vacuum Einstein equation to second order we must correct $\gamma^{(1)}{}_{\mu\nu}$ by adding to it the term $\gamma^{(2)}{}_{\mu\nu}$, where $\gamma^{(2)}{}_{\mu\nu}$ satisfies

$$G^{(1)}_{\mu\nu}[\gamma^{(2)}{}_{\alpha\beta}] + G^{(2)}_{\mu\nu}[\gamma_{\alpha\beta}] = 0, \quad \text{where} \quad \gamma_{\mu\nu} = \gamma^{(1)}{}_{\mu\nu} + \gamma^{(2)}{}_{\mu\nu} \tag{11}$$

which is the correct form of Equation (4.4.52) in Wald's book, where he did not distinguish $\gamma_{\mu\nu}$ from $\gamma^{(1)}{}_{\mu\nu}$.

This equation does have a solution for the static case. However, detailed calculation shows that this equation does not have a solution for the dynamic case [7-9]. The fact that, for a dynamic case, there is no bounded solution for Equation (11) means also that the Einstein equation does not have a dynamic solution. The known examples are the metric of Bondi et al. [20] and the metric of Misner et al. [13]. The principle of causality requires the existence of a dynamic solution, but Wald did not see that the Einstein equation can fail this requirement [24].

Misner et al. [13] also believed that the Einstein equation has dynamic solutions. They claimed that there is a bounded dynamic metric form as follows,

$$ds^2 = c^2dt^2 - dz^2 - L^2\left(e^{2\beta}dx^2 + e^{-2\beta}dy^2\right), \tag{12}$$

where $L = L(u)$, $\beta = \beta(u)$, $u = ct - z$, and c is the light speed. Then, the Einstein equation $G_{\mu\nu} = 0$ becomes

$$\frac{d^2L}{du^2} + L\left(\frac{d\beta}{du}\right)^2 = 0. \tag{13}$$

Such a claim is clearly in conflict with the non-existence of dynamic solutions.

Apparently, they incorrectly believe this is a case different from the metric of Bondi et al. [20]. However, such a claim is due to a blind faith on Einstein's claim on the existence of the dynamic solution together with a careless calculation at the undergraduate level.

They further claimed [13],

"The linearized version of $L'' = 0$ since $(\beta')^2$ is a second-order quantity.

Therefore the solution corresponding to Linearized theory is

$$L = 1, \ \beta(u) \text{ arbitrary but small.} \tag{14}$$

The corresponding metric is

$$ds^2 = (1 + 2\beta)dx^2 + (1 - 2\beta)dy^2 + dz^2 - dt^2, \qquad \beta = \beta(t - z).\text{"} \tag{15}$$

However, careful calculation shows that these claims are also incorrect. In other words, Misner et al. [13] are incorrect and Equation (13) does not have a solution that satisfies Einstein's requirement on weak gravity.

In fact, $L(u)$ is unbounded even for a very small $\beta(u)$. Linearization of (13) yields $L'' = 0$, and this leads to $\beta'(u) = 0$. Thus, this leads to a solution $L = C_1\mu + C_2$, where C_1 and C_2 are constants. Therefore, the requirement $L \approx 1$ implies $C_1 = 0$. However, $\beta'(u) = 0$ implies no waves. Thus, metric (15) cannot be derived from the Einstein equation.

To prove Equation (13) having no wave solution, it is sufficient to consider the case of weak gravity. According to Einstein, for weak gravity of metric (12), one would have

$$L^4 \cong 1, \qquad e^{\pm 2\beta} \cong 1 \qquad \text{and} \qquad L(u) >> |\beta(u)|. \tag{16}$$

Since $L(u)$ is bounded, $L'(u)$ cannot be a monotonic function of *u*, unless $L' - > 0$. Thus, there is an interval of *u* such that the average,

$$\langle L'' \rangle = 0. \tag{17}$$

However, according to Equation (13) this average is larger than zero unless $\beta'(u) = 0$ in the whole interval of *u*.

Also, from Equation (16), one would obtain $L(\cong 1) > 0$, and one has $0 > L''(u)$ if $\beta'(u) \neq 0$. Thus, $-L'(u)$ is a monotonic increasing function in any finite interval of *u* since $\beta'(u) = 0$ means $L'' = 0$, i.e., no wave. In turn, since $\beta'(u)$ is a "wave factor", this implies that $L(u)$ is an unbounded function of *u*. Therefore, this would contradict the requirement that *L* is bounded. In other words, Equation (13) does not have a bounded wave solution. Moreover, the second order term L'' would give a very large term to L, after integration.

Now, let us investigate the errors of Misner et al. [13; p. 958]. They assumed that the signal $\beta(u)$ has duration of $2T$. For simplicity, it is assumed that definitely $|\beta'(u)| = \delta$ in the period $2T$. However, detailed calculation shows that independent of the smallness of $2\delta^2 T$ (or details of $|\beta'(u)|^2$), Equation (12) has no weak wave solution. Moreover, $|L(u)|$ is not bounded since it would become very large as u increases. Thus, restriction of $2\delta^2 T$ being small [13] does not help.

Thus, one cannot get a wave solution through linearization of Equation (13), which has no bounded solution. The assumption of metric form (12) is bounded, and has a weak form (15), is not valid. Thus, there is no bounded wave solution for the non-linear Einstein equation, which violates the principle of causality [25].

The root of their errors was that they incorrectly assumed that a linearization of the Einstein non-linear equation would produce a valid approximation. Thus, they implicitly and incorrectly assume the existence of a bounded wave solution without the necessary verification, and thus obtain incorrect conclusions.

On the other hand, from the linearization of the Einstein equation (the Maxwell-Newton approximation) in vacuum, Einstein [26] independently obtained a solution as follows:

$$ds^2 = c^2 dt^2 - dz^2 - (1 + 2\phi)\, dx^2 - (1 - 2\phi)\, dy^2, \tag{18}$$

where ϕ is a bounded function of $u (= ct - z)$. Note that metric (18) is the linearization of metric (12) if $\phi = \beta(u)$, but it cannot be obtained through the non-linear Einstein equation.

Thus, the problem of waves illustrates that the linearization is not valid for the dynamic case when gravitational waves are involved since Equation (13) does not have a bounded wave solution. In other words, the Einstein equation and its linearization are independent equations. Moreover, there are other errors made by Misner et al. [13]. For instance, in disagreement to Weinberg [27], they were unable to rectify their error on local time shown in their Equation (40.14).

There are others who also make errors on the issue of dynamic solutions.

Christodoulou and Klainerman [15], and ‘t Hooft [16] claimed to have examples of bounded dynamic solutions. However, these are also due to errors in calculation and/or misconceptions as the case of Misner et al. [13]. Christodoulou and Klainerman [15] claimed to have constructed dynamic solutions, but their construction is actually incomplete [28]. Moreover, Taylor of Princeton was unable to justify their calculation on the binary pulsar experiments [17] when P. Morrison of MIT questioned him [29].

In defense of the errors of the Nobel Committee for Physics of 1993, ‘t Hooft [16, 30] comes up with a bounded time-dependent cylindrical symmetric solution as follows:

$$\Psi(r, t) = A\int_0^{2\pi} d\varphi e^{-\alpha(t - r\cos\varphi)^2}, \tag{19}$$

where A and $\alpha(> 0)$ are free parameters. $|\Psi|$ is everywhere bounded. However, it has been shown that there are no valid sources that can be related to solution (19) [30]. Thus, since the principle of causality is also violated, his solution is not valid in physics.

6. The Misleading Positive Mass Theorem of Yau and Schoen

If physicists made errors because of inadequacy in mathematics, one might expected that mathematicians would not have such a problem. However, the mathematicians may have the problem of not understanding the physics. Thus, although they may not commit mistakes in mathematics, their conclusion can be misleading because of using the invalid assumption in physics. A good example is the positive mass theorem of Schoen and Yau in general relativity.

In general relativity, the positive mass theorem [18, 19] of Schoen and Yau states that, assuming the dominant energy condition, the mass of an asymptotically flat spacetime is non-negative; furthermore, the mass is zero only for Minkowski spacetime. The free encyclopedia Wikipedia even summarizes the contributions of Prof. Yau as follows:

“Yau’s contributions have had a significant impact on both physics and mathematics. Calabi-Yau manifolds are among the ‘standard tool kit’ for string theorists today. He has been active at the interface between geometry and theoretical physics. His proof of the positive energy theorem in general relativity demonstrated - sixty years after its discovery - that Einstein's theory is consistent and stable. His proof of the Calabi conjecture allowed physicists - using Calabi-Yau compactification - to show that string theory is a viable candidate for a unified theory of nature."

Thus, the positive energy Theorem [18, 19] would have profound influence that leads to even the large research efforts on string theory. Moreover, it was claimed that Einstein's theory is consistent and stable. This is in a direct conflict with the fact that there is no dynamic solution for the Einstein equation.

In the positive mass theorem, an implicit assumption is that all the coupling constants have the same sign. Then, it is crucial to consider the assumption of asymptotically flat spacetime, which can be considered as common since it is satisfied in stable solutions such as the Schwarzschild solution, the Kerr solution, etc. Thus, it could be "natural" to assert (as in Wikipedia) that Einstein's theory is consistent and stable.

However, if one understands the physics in general relativity as well as Gullstrand [5] does, the above statement is clearly incorrect. It has been proven that the Einstein equation has no dynamic solution, which is bounded [7-9]. Thus, the assumption of asymptotically flat implies the exclusion of the most important class of solutions, the dynamic solutions. Therefore, Schoen and Yau actually prove a trivial result that the total mass of a static (or stable) solution is positive. In other words, the conclusions drawn from the positive theorem are grossly misleading. This illustrates that an inadequate understanding in physics can lead to beautiful, but actually completely invalid statements in physics.

The condition of asymptotically flat is a normal condition in physics and thus, for a valid theory, it should not exclude the case of a dynamic solution. The problem rises from the Einstein equation that does not have a bounded dynamic solution as Gullstrand suspected [5]. Thus, the problem is due to that Yau and Schoen used an implicit assumption, the existence of bounded dynamic solutions, which is actually false but was not stated in their theorem. Nevertheless, Atiyah, being a pure mathematician, was not aware of the problem of non-existence of bounded dynamic solutions. Thus, one should not be surprised that such an error was made twice over eight years (1982-1990) by the Fields medal. Note that the proof for the nonexistence of a bounded dynamic solution was published in 2000 [9].

Moreover, the necessity of modifying the Einstein equation is not the only problem for the case of massive particles. Another problem is that there is no radiation reaction force. In a geodesic equation,

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0, \text{ where}$$

$$\Gamma^\mu\alpha\beta = (\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta})g^{\mu\nu}/2 \tag{20}$$

and $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, the force is generated by metric elements, which are unrelated to the test particle. Also, when charged particles are involved, there is a new charge-mass interaction that general relativity cannot accommodate [24]. Thus, it is very misleading to claim that Einstein's theory is self-consistent.

7. Unification of Gravitation and Electromagnetism

Now, let us consider the case when electromagnetism is involved. Since the photons include gravitational energy, the unification of gravitation and electromagnetism is necessary. In general relativity, it is clear that a charge can create a field that couple with a mass. This is shown by the Reissner-Nordstrom metric [13] (with $c = 1$) created by a charged particle with mass M is as follows:

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2. \tag{21}$$

However, it is not clear that a mass can create a field that couples with a charge.

The static force that acts on a test particle P with mass m for the first order approximation is

$$-m\frac{M}{r^2} + m\frac{q^2}{r^3} \tag{22}$$

since $g^{rr} \cong -1$. Note that it is clear that the second term is a repulsive force due to the static charge-mass interaction [31]. According to the reaction force is equal to but in the opposite direction of the acting force, the test particle P must create a field m/r^3 that couples to q^2. This would mean that unification between electromagnetism and gravitation is necessary [24]. However, the newly discovered force that should have been discovered in 1916 was over-looked until 1997 [32].

This was so because of two misconceptions: 1) Gravity is always attractive; 2) $E = mc^2$ was incorrectly considered as unconditional. The non-existence of a dynamic solution for the Einstein equation leads to the discovery that there must be different coupling signs for the dynamic case [7]. This non-uniqueness of couplings leads to the investigation that the charge-mass interaction is discovered [7-9].

The experiments on a charged ball confirm the repulsive charge-mass interaction [33], and this would confirm the unification of electromagnetism and gravitation. Einstein overlooked the coupling of charge square in the five-dimensional theory [34] because he believed, unlike Maxwell, that a new interaction should not be created. In other words, Einstein has over confidence on general relativity. Since formula (22) is generated by general relativity and thus is also a test for general relativity.

However, the charge square coupling is clearly beyond Einstein's general relativity. This is a problem of unification between electromagnetism and gravitation [33].

8. Conclusions and Discussions

Now, the most important conclusions from this paper are: 1) $E = mc^2$ is only conditionally valid. In particular, the electromagnetic energy is not equivalent to mass. 2) To be consistent with $E = mc^2$, the photons must include energy from its gravitational components. 3) Einstein's general relativity is valid only for the static and stable cases, but is invalid for the dynamic case, for which it remains to be rectified and completed in at least two aspects: 1) The exact form of the gravitational energy-stress tensor is not known; and 2) The radiation reaction force is also not known. Note that, considering the radiation reaction force, general relativity as a theory of geometry is invalid. Moreover, since the photons include gravitational energy, the unification of gravitation and electromagnetism is necessary. To this end, a potentially very strong candidate is a five-dimensional theory.

Moreover, the existence of a dynamic solution requires, as shown in Equation (6), an additional gravitational energy-momentum tensor with an antigravity coupling. Thus, the space-time singularity theorems, which require the same sign for couplings, are actually irrelevant to physics. The positive energy theorem of Schoen and Yau is only for stable solutions because their theorem actually does not include the dynamic solutions. Moreover, $E = mc^2$ is only conditionally valid, and such recognition is crucial to identify the charge-mass interaction. This repulsive force also can potentially explain the Space-Probe Pioneer anomaly [33]. This force would also be useful to detect things underground and/or under water since the strength of such detection can be adjusted with the potential of a charged capacitor [33]. Experimentally, in contrast to the claim of Einstein [2] based on $E = mc^2$, a piece of heated-up metal would have reduced weight [35]. Moreover, since such a force is coupled to the charge square, such a force exists in a five-dimensional theory. Thus, a study of a five-dimensional theory of five variables is strongly recommended.

The experimental confirmation of such an interaction means that Einstein's unification between electromagnetism and gravitation is proven valid [33]. Einstein failed to show such unification because of his two shortcomings: 1) He failed to see as Maxwell [36] did that unification is necessary to create new interactions. 2) He has mistaken that $E = mc^2$ was unconditional. It was a surprise that his famous formula $E = mc^2$ is only conditionally valid. This misunderstanding is the main cause of his failure to establish the unification of gravitation and electromagnetism.

Subsequently, this misunderstanding has also led to the incorrect beliefs that all the coupling constants have the same sign and the existence of dynamic solutions for the Einstein equation. It should be noted that the misinterpretation of $E = mc^2$ as unconditional is a popular error. Even Nobel Laureates such as 't Hooft and Wilczek made such errors in their Nobel speeches [37, 38].

In conclusion, Einstein is still the number one theoretical physicist after the rectification of his theories. Moreover, the charge-mass interaction is present, but neglected in many microscopic problems. Thus, Einstein's view that quantum mechanics is not a final theory now has new evidence.

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