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RULER MEASUREMENTS GIVE SPACE-TIME-TRANSFORMATION-INDEPENDENT INVARIANT LENGTHS

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Abstract

Two thought experiments are described in which ruler measurements of spatial intervals are performed in different reference frames. They demonstrate that such intervals are frame-independent as well as independent of the nature of the space-time transformation equations. As explained in detail elsewhere, the 'length contraction' effect of conventional special relativity theory is therefore spurious and unphysical.

1. Introduction

The concept of a 'ruler measurement' of a spatial interval is a very familiar one in the everyday world. A 'ruler' is a flat object with a rectilinear boundary furnished with equally spaced 'marks', M, specifying positions along the boundary, labeled by ordinal numbers: $M(I)$, $I = 1, 2, 3, \ldots$. In order to perform a length measurement, 'pointers' on the objects, the spatial separation of which is to be determined, are placed against the ruler and the closest marks to the pointers $M(I), M(J)$ $(J > I)$ are noted. The ruler measurement of the spatial separation of the pointers is then $J - I$ in units of the inter-mark separation, with a maximum uncertainty of one unit.

Keywords and phrases: special relativity, ruler measurements, translational invariance, invariant length intervals.

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A mathematical calculus to rigorously specify such measurements, in terms of pointer-mark coincidences (PMC), was proposed in [1], but for the purposes of the present paper the simple definition given above is sufficient.

In the case that the to-be-measured objects are at rest relative to the ruler, time plays no role in the measurement. However, if they are in uniform or accelerated motion, with the same or different velocities parallel to the edge of the ruler, the times at which the PMCs, constituting the raw measurements, are observed are important. Two distinct situations are possible:

(1) The objects have different velocities.

(2) The objects have the same (possibly time-varying) velocity at all times.

In the first case, the spatial interval between the objects changes with time and a measurement of the spatial separation at any epoch[1] requires the simultaneous recording of the two PMCs. In case (2), which represents the examples to be discussed in this paper, simultaneous observation of the PMCs is also required to define the spatial separation of the objects, but the latter is a time-independent quantity. Note that the spatial separation in case (2) is a constant, independent of both time and the velocity of the objects, provided that the latter is the same at all epochs. This statement is valid for both uniform and accelerated motion of the two objects.

Using the above definitions it will now be shown, using two simple examples, that ruler measurements of the spatial separation of two objects undergoing similar motion give the same result in both an inertial frame from which their motion is observed, or in the co-moving reference frame of the two objects. Furthermore, this equality is independent of the transformation equations relating space and time coordinates in the two frames. A corollary is that the ‘length contraction’ and ‘relativity of simultaneity’ effects of conventional special relativity are illusory [2, 3, 4, 5].

The first example is the thought experiment shown in Figure 1. Two ‘pointer trams’, PT1 and PT2, are at rest on straight tracks aligned with two mark poles M3

[1]The word ‘epoch’ denotes a particular instant of time in some reference frame. It can be operationally defined as a particular PMC corresponding to a spatial coincidence between the moving hand of an analogue clock at rest in the frame (the pointer) and a mark on its dial.

and M4, respectively, separated by the distance L. The ruler measurement of the initial separation of the trams is thus L. Two further mark poles, M1 and M2, are displaced from M3 and M4, respectively, by a distance $3L$ in the direction of motion of the trams. All of the mark poles are equipped with lamps. Mark poles M3 and M4 are also equipped with local synchronized clocks connected to the lamps and the system controlling the motion of the adjacent tram. Because the clocks are at rest in the same inertial frame, they may be synchronized by any convenient method; for example, by 'pointer transport' as described in [1], or by clock transport from a suitably placed master clock equidistant from M3 and M4. On the assumption of light speed isotropy in the frame of the mark poles the clocks may also be synchronized by exchange of light signals, according to the Einstein procedure [6]. Even without assuming light speed isotropy, the clocks may be externally synchronized by reception of essentially parallel-moving light signals transmitted simultaneously by a distant source equidistant from M3 and M4. In this case the clocks are stopped with the same setting, and are started on receipt of the signals. Since the clocks and the signal source are at rest in the same frame during the synchronization procedure there is no necessity to consider any putative 'relativity of simultaneity' which may occur when synchronized clocks are observed in different inertial frames.

The lamps attached to M3 and M4 flash when the trams start to move, in an identical manner, down the tracks while those attached to M1 and M2 flash at the epoch when the front ends of the moving trams are aligned with them. The simultaneity of the signals of the lamps on M1 and M2 or M3 and M4 endorses the validity of the corresponding ruler measurements.

Configurations observed in the rest frame, S, of the mark poles are shown in Figures 1a and 1b. At $t = 0$, as recorded by a clock at rest in S, synchronous signals from the local clocks at M3 and M4 cause the lamps at M3 and M4 to flash and PT1 and PT2 to start to move towards M1 and M2. At time $t = T$, when the instantaneous velocity of PT1 and PT2 is $v(T)$, the front of PT1 is aligned with M1 and the front of PT2 is aligned with M2. At this instant the lamps on M1 and M2 flash. The measured spatial separation of the two trams is then L, the same as when they were at rest, demonstrating the time-independence of their separation for case (2).

The same journey of the trams between the mark poles, as observed in the co-moving frame S' of the trams, is shown in Figures 1a and 1c. If t' is the epoch recorded by a clock at rest in the frame S', then at $t = t' = 0$, S' is identical to S, so the configuration is again that of Figure 1a. Observers at rest in the trams will see the mark poles M1 and M2 start to move simultaneously towards them, with identical motion, so that at the time $t' = T'$, when their observed velocity is $v'(T')$, they arrive simultaneously at the front ends of PT1 and PT2, respectively, when their lamps flash (Figure 1c). For this ruler measurement, showing that the separation of M1 and M2, as measured in S', is also L, the roles of the trams and mark poles are inverted. The ends of the trams constitute the marks and the moving poles the pointers. It is evident that the flashes of the lamps on M1 and M2 are simultaneous in both S and S' - there is no 'relativity of simultaneity' effect.

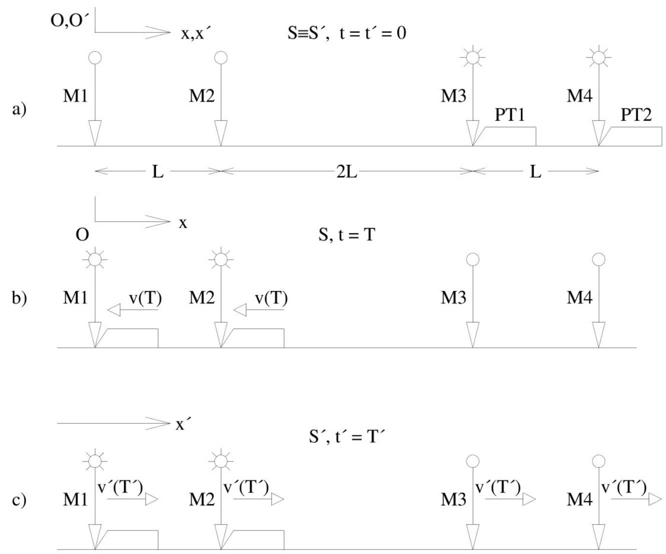

Figure 1. Spatial configurations in frames S [(a), (b)] and S' [(a), (c)] of a thought experiment in which the pointer trams PT1 and PT2 are simultaneously accelerated in a similar manner so as to pass from positions of spatial coincidence with stationary mark poles M3 and M4 at $t = t' = 0$ [(a)] to simultaneous spatial coincidence with M1 and M2 at time T in S [(b)] or time T' in S' [(c)]. Time coincident events are signaled by simultaneous flashes of lamps on M3 and M4 in (a) and on M1 and M2 in (b) and (c). See text for discussion.

The conclusions of the thought experiment - the invariance of the results of ruler measurements of the spatial separations of PT1 and PT2 or M1 and M2 performed in different frames in relative motion and the absence of any relativity of simultaneity effect - follow only from the initial postulate of identical motion of the trams in the frame S, and are independent of the form of the space-time transformation equations relating t' to t. Suppose the acceleration of PT1 or PT2 is some arbitrary function of $t : a(t)$. The velocity at epoch t of PT1 or PT2 is then

$$v(t) = \int_0^t a(t')dt' \tag{1}$$

and the spatial displacement, $d(t)$, at epoch t is

$$d(t) = \int_0^t v(t'')dt'' = \int_0^t dt'' \int_0^{t''} a(t')dt'. \tag{2}$$

The spatial separations $\Delta x_{PT}(t)$ and $\Delta x'_{PT}(t')$ in S and S$'$, respectively are, from the geometry of Figure 1

$$\begin{aligned} \Delta x_{PT}(t) &\equiv x(PT2) - x(PT1) \\ &= [4L - d(t)] - [3L - d(t)] \\ &= L = \Delta x_{PT}(0) = \Delta x'_{PT}(t'). \end{aligned} \tag{3}$$

Space-time transformation equations yield from $a(t)$ the corresponding acceleration $a'(t')$ of M1 and M2 as observed in the frame S$'$. Similarly for (1) and (2) above, it follows that

$$v'(t') = \int_0^{t'} a'(t''')dt''', \tag{4}$$

$$d'(t') = \int_0^{t'} v'(t'')dt'' = \int_0^{t'} dt'' \int_0^{t''} a'(t''')dt'''. \tag{5}$$

The spatial separations $\Delta x'_M(t)$ and $\Delta x_M(t)$ of M1 and M2 in S$'$ and S are

$$\begin{aligned} \Delta x'_M(t') &\equiv x'(M2) - x'(M1) \\ &= [L + d'(t')] - d'(t') \\ &= L = \Delta x'_M(0) = \Delta x_M(t). \end{aligned} \tag{6}$$

The world lines of PT1, PT2, M1 and M2 in S and S′ for different acceleration programs and space-time transformation equations are shown in Figure 2 for the following examples

(i) 'Parabolic motion' in special relativity [7, 8, 9, 10, 11]

$$x_{PT1}(t) = x_{PT2}(t) - L = 3L - \frac{c^2}{a_0}\left[\sqrt{1+\left(\frac{a_0 t}{c}\right)^2} - 1\right], \tag{7}$$

$$x'_{M1}(t') = x'_{M2}(t') - L = \frac{c^2}{a_0}\left[\cosh\frac{a_0 t'}{c} - 1\right]. \tag{8}$$

(ii) Constant acceleration in Galilean relativity

$$x_{PT1}(t) = x_{PT2}(t) - L = 3L - \frac{1}{2}a_0 t^2, \tag{9}$$

$$x'_{M1}(t) = x'_{M2}(t) - L = \frac{1}{2}a_0 t'^2. \tag{10}$$

These world lines are the $c \to \infty$ limits of (7) and (8)

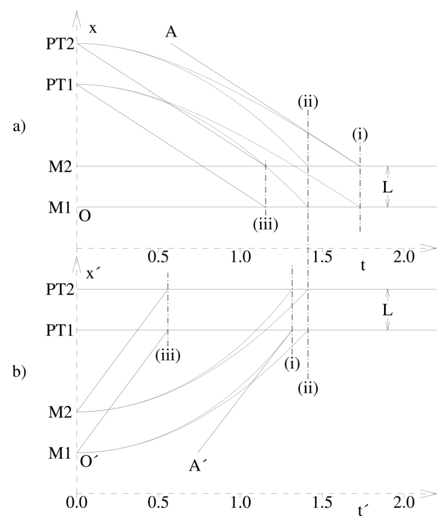

Figure 2. World lines of PT1, PT2, M1 and M2 in S (a) or S′, (b) for different acceleration programs and space-time transformation equations. $c = a_0 = 1$, $L = 1/3$. Ruler measurements of the spatial separations of PT1 and PT2 or M1 and M2 are indicated by the vertical dot-dashed lines for the three different cases considered. See text for discussion.

(iii) Uniform motion after impulsive acceleration in special relativity

$$x_{PT1}(t) = x_{PT2}(t) - L = 3L - vt, \tag{11}$$

$$x'_{M1}(t) = x'_{M2}(t) - L = v't' = \gamma vt, \tag{12}$$

where $\gamma \equiv \sqrt{1-(v/c)^2}$. The constant velocity, v, is chosen according to the equations [11]:

$$T = \frac{c}{a_0}\sqrt{\left(1+\frac{3a_0L}{c}\right)-1}, \tag{13}$$

$$v = -\frac{a_0T}{\sqrt{1+\left(\frac{a_0T}{c}\right)^2}} = -\frac{a_0T}{\gamma(T)}, \tag{14}$$

$$v' = c\sinh\frac{a_0T'}{c} = a_0T = -\gamma(T)v(T). \tag{15}$$

In this way, PT1 and PT2 have the same velocity after impulsive acceleration as they have when arriving at M1 and M2 in case (i) above. It corresponds to the limits $a_0 \to \infty$, $T \to 0$ in Equations (14) and (15) for finite values of a_0T, v and v'.

Since the $c \to \infty$ (Galilean) and $t, t' \to 0$ limit of (7) and (8) are the same, for the choice of units and parameters $c = a_0 = 1$, $L = 1/3$ in Figure 2, the world lines of PT1, PT2 in S (Figure 2a) and of M1, M2 in S' (Figure 2b) are indistinguishable, for $t, t' < 0.5$ between cases (i) and (ii). For case (ii) the shapes of the world lines of M1 and M2 in S' are mirror images of those of PT1 or PT2 in S. The different shapes of the world lines of M1 and M2 in S' to those of PT1 or PT2 in S for case (i) are due to the time dilation effect. In fact, at corresponding values of t and t', the slopes of the world lines of M1 and M2 in S' are γ times the slopes of those of PT1 or PT2 in S [11]. The straight world lines of PT1 or PT2 in S and of M1 and M2 in S' for case (iii) have, due to the time dilation effect, slopes in the ratio $1:\gamma$, where $\gamma = 2$ corresponding to $v = -c\sqrt{3}/2$ or $c = 1$, $a_0T = \sqrt{3}$ in Equations (14) and (15).

The lines A (A'), which are tangential to the world lines of PT2 in Figure 2a and M1 in Figure 2b, for case (i), where they intersect those of M2 and PT1, respectively, are parallel to the world lines of PT1 or PT2 (M1 or M2) for case (iii).

The spatial separation of PT1 and PT2 or M1 and M2 is equal to *L* at all times in both S and S'. Ruler measurements of this separation for the three different cases in both frames are indicated by the labeled vertical dot-dashed lines. It can be seen in Figure 2 that the constancy and frame invariance of the spatial separations of PT1 and PT2 or M1 and M2 is a necessary geometrical consequence of the identical shapes of their world lines in each frame in all cases. This identity of shape is, in turn, a necessary consequence of the identical nature of the acceleration programs to which they are subjected. The invariance of the separation is also independent of the form of the space-time transformation equations by which the shape of the world lines in S' may be derived from those in S.

The initial configuration of the second thought experiment is shown in Figure 3a. The measuring rod MR, of length *L*, is used to set the separations of the ruler-mark objects A_1, B_1, A_2, B_2, A_3 and B_3 of the ruler R. The separations of A_1 - B_1, B_1 - A_2, A_2 - B_2 and A_3 - B_3 are set to *L*, and that of B_2 - A_3 to 2*L*. The front and back ends of MR (as viewed from the left side of the ruler in Figure 3a) are denoted by F and B, respectively. As discussed above, if the ends of the moving object are simultaneously aligned with any two of the mark-objects in the proper frame of the ruler, the length of the moving object is defined to be equal to the separation of the mark-objects in the proper frame of the ruler.

The ruler R is subjected to an acceleration program similar to that in case (i) above, specified by the parameter a_0, in such a sense that, in the proper frame, S' of R, MR is observed to move to the right. The equations describing the world lines of the ends of MR as observed in S', are then similar to (8) above:

$$x'(F) = x'(B) - L = \frac{c^2}{a_0}\left[\cosh\frac{a_0 t'}{c} - 1\right]. \qquad (16)$$

In Figure 3a, the length of MR at rest, *L*, is measured by the separation of A_1 and B_1. It follows from (16) that the length of MR is also measured to be *L* by simultaneous coincidence of F and R with B_1 and A_2 at the epoch $t'_1 = 0.693\,c/a_0$. In Figures 3-5, units and dimensions are chosen so that $a_0 = c = 4L = 1$. Similar measurements are performed by A_2 and B_2 at $t'_2 = 0.962\,c/a_0$ and by A_3 and B_3 at $t'_3 = 1.46c/a_0$. As in [11], it is assumed that the acceleration halts when $x'(F) = c^2/a_0$ at the epoch in S':

$$t'_{acc} = \frac{c}{a_0}\operatorname{arccosh}(2) = 1.317\,\frac{c}{a_0} \tag{17}$$

so that for epochs $t' \geq t'_{acc}$, S' is an inertial frame and v' is constant.

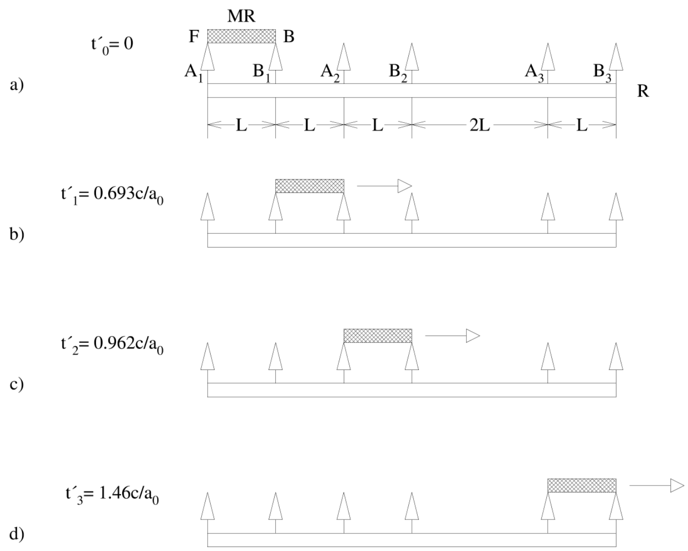

Figure 3. Four measurements of a stationary measuring rod MR are performed by a moving ruler R. The measurements as observed in the proper frame, S', of R are shown. In (a), MR is at rest relative to R. In (b) and (c) measurements are made while R is accelerating. In (d) the ruler moves uniformly relative to MR. In all cases MR is measured to have the same length *L* - there is no 'length contraction' effect.

The world lines of F and B in S' and the four measurements of the length of MR - one at rest, two during accelerated motion and one during uniform motion - are plotted in Figure 4. Again, each concordant ruler measurement is indicated by a vertical dot-dashed line. This figure shows clearly that the physical basis for the equality of all the length measurements is that the first member of (16) may be transposed to give:

$$x'(B, t') - x'(F, t') = L, \tag{18}$$

where the t' dependence of each term is shown explicitly. Thus the world line of B is derived from that of F by displacing it with the distance L along the positive x'-axis. The operation $x' \to x' + L$ corresponds to moving the origin of the coordinates by a distance L. The shape of the world line of one end of MR is invariant under this transformation, and so manifests translational invariance.

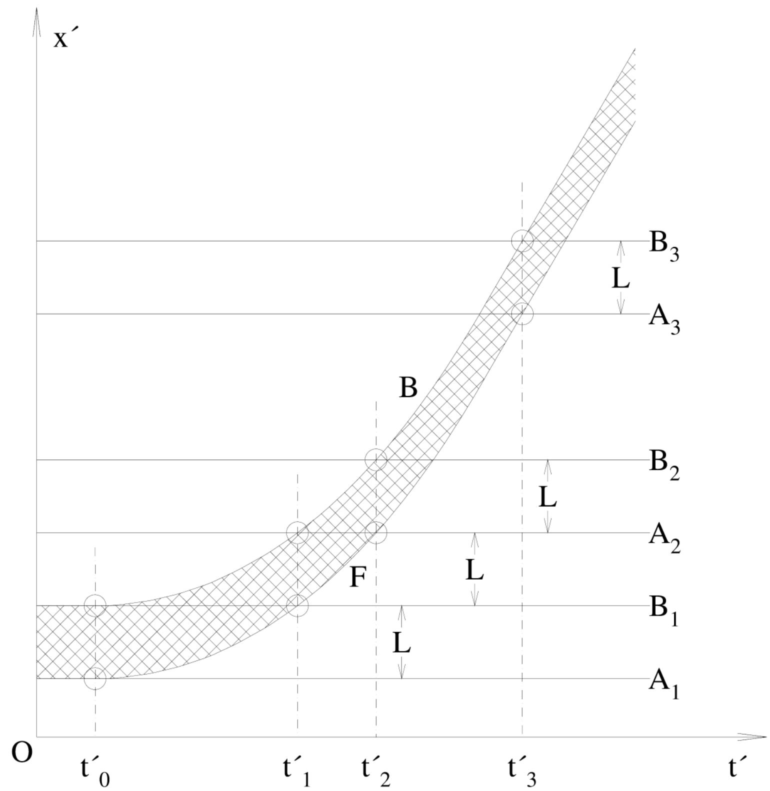

Figure 4. World lines of the front (F) and back (B) ends of the stationary measuring rod MR, as viewed from the proper frame of the ruler R, showing the epochs t'_0, t'_1, t'_2 and t'_3 of the four concordant measurements of the length of MR as indicated by the vertical dot-dashed lines.

For definiteness, the case of 'hyperbolic motion' of the ruler, according to Equation (16), was considered above, but it is clear that the invariance of length measurements of MR must be independent of the acceleration program of R. In Figure 5, for example, a series of impulsive accelerations of sign $+, -, +, -, +$ are applied, yielding equal length measurements at the epochs t'_0, t'_1, t'_2, t'_3 and t'_4 as shown. One measurement corresponds to a simultaneous coincidence of F - A_1 and B - B_1, three to simultaneous coincidences of F - A_2 and B - B_2, and one to a simultaneous coincidence of F - A_3 and B - B_3.

Discussion of the reason for the spurious nature of the correlated 'length contraction' and 'relativity of simultaneity' effects of conventional special relativity [6] may be found in [1, 2, 3, 4, 5]. In particular it is shown in [2], that these effects result from the neglect of certain additive constants in the space-time Lorentz transformation equations, mentioned by Einstein [6], but not implemented by him, that are required to correctly describe synchronized clocks at different spatial positions.

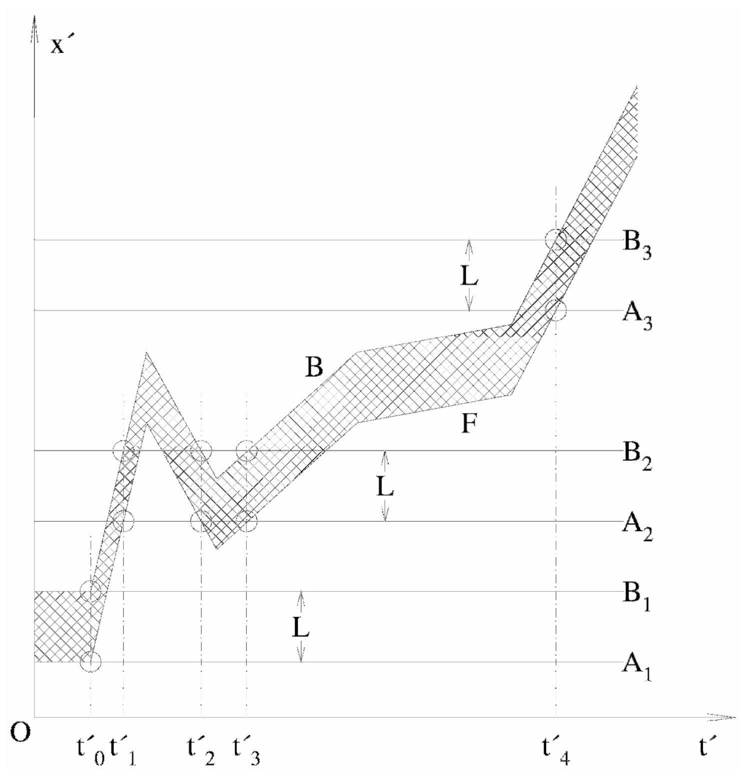

Figure 5. World lines of the front (F) and back (B) ends of the stationary measuring rod MR, as viewed from the proper frame of the ruler R, when the latter is subjected to a series of impulsive accelerations. Five concordant measurements of the length of MR, indicated by the vertical dot-dashed lines, at epochs t'_0, t'_1, t'_2, t'_3 and t'_4 are obtained.

Acknowledgements

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