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FRACTAL PHYSICS THEORY - FOUNDATION

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Abstract

This first article, in a series of five, introduces Fractal Physics Theory, which is founded on extending the two Special Relativity postulates to scale. The first postulate assumes absolute uniform scale cannot be detected. The meter, kilogram, and second relate to a person's size, mass, and awareness of time passage, the human scale. Atoms are tiny and stars are enormous, both relative to the human scale. In addition to obvious size and mass-energy differences, there also exists a time scale difference between the quantum and cosmic scales as viewed from the human scale. Quantum scale objects appear to exist in accelerated time frames, while cosmic scale objects appear to exist in decelerated time frames, both relative to the human scale. The second postulate assumes the speed (c) of electromagnetic radiation photons in vacuum is independent of scale. This series of articles is intended to call attention to the remarkable self-similarity that exists between the quantum and cosmic scales.

1. Introduction

The applicability of Fractal Geometry to further understanding of diverse phenomena has proliferated since Benoit Mandelbrot's seminal work in the 1970s [1]. Special Relativity postulates generalized to "scale" are the basis of the two postulates of Fractal Physics Theory. Laurent Nottale's Scale Relativity Theory generalizes to scale only one Special Relativity postulate [2]. The works of Mandelbrot and Nottale inspired the development of Fractal Physics Theory (FPT).

Few years ago the author became acquainted with the efforts of a relatively small

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group of theorists grappling with incorporating the mathematics of the infinite into the physics of our Universe. The E-infinity group appears to be the first to apply the theory of fractal-Cantorian spacetime to physics, where spacetime is an infinite-dimensional, hierarchical, disconnected non-differentiable Cantor set. It is free of quantum field theory anomalies and is inherently scale invariant because it is geometry without ordinary points. The theory has found success in calculating coupling constants and the mass spectrum of elementary particles [3-5]. Fractal Physics Theory is founded on two postulates, however, E-infinity Theory was not postulated, it was derived mathematically from first principles using set theory. If both theories truly depict physical reality, then they must be complimentary.

As discussed by Marek-Crnjac [6], analytical continuation converts an ordinary diffusion equation into the Schrödinger equation and also converts a telegraph equation into a Dirac equation. Analytical continuation is a short cut to quantization, and replaces time t with imaginary time it, where $i = \sqrt{-1}$. In 1983 Garnet Ord showed that analytical continuation is not required if fractal space-time is used. It is transfinite geometry and not quantization which produces the equations of quantum mechanics. The "ensembles of classical point particles" used successfully by Ord and Mann [7] can be attributed to subquantum scale atoms of FPT. In 1990 El Naschie showed that the n-dimensional triadic Cantor set had the same Hausdorff dimension as the dimension of a random inverse golden mean Sierpinski space to the power $n-1$ [6].

The Cantor set is the simplest transfinite set that exists. The triadic Cantor set is a point of measure zero, with Hausdorff dimension $\ln 2/\ln 3 \sim 0.6309$. The Cantor set has an infinite number of points (a discrete structure) but has the exact number of points as in its initial line continuum (same cardinality). Mauldin and Williams constructed a one dimensional golden mean Cantor set. They used a uniform probabilistic distribution to introduce randomness. Space filling means no gaps and no overlapping and introduces randomness. The crucial difference between the normal fractal structure of the Cantor set and the E-infinity Cantorian spacetime is the introduction of randomness for optimal density. The golden mean is the most irrational number that can exist due to the fact it is the least efficient number approximated by a rational number. The topological dimension is always an integer value. The Menger-Urysohn dimension generalizes the topological dimension by setting the dimension zero for all Cantor sets and –1 as the empty set dimension [3-5].

The bijection formula is central to E-infinity spacetime [3-5]:

$$d_c{}^{(n)} = (1/d_c{}^{(0)})^{n-1}, \tag{1}$$

$d_c{}^{(0)} =$ Hausdorff dimension of a single elementary Cantor set in one topological dimension. From the Mauldin-Williams theorem, a one dimensional randomly constructed Cantor set will have the Hausdorff dimension equal to the golden mean $\varphi = 0.5(\sqrt{5}-1) \sim 0.618034 = d_c{}^{(0)}$,

$n =$ Menger-Urysohn dimension.

E-infinity established a non-differentiable and non-metric topological space described by three dimensions:

(1) The spacetime manifold is formally infinite dimensional.

(2) A finite average "expectation value" Hausdorff dimension $= 4 + \varphi^3 \approx 4.236068$.

(3) A finite topological Menger-Urysohn dimension $= 4$.

Looking at the manifold from a great distance, by reducing the accuracy of the observation, this manifold appears smooth and differentiable with the four dimensions of relativity space-time. Magnifying a section of this manifold reveals its non-differentiable structure. Consequently, this manifold appears to have different dimensions based on the observation resolution. With continued magnification a four dimensional manifold is seen surrounding another four dimensional manifold which surrounds another four dimensional manifold, ad infinitum [3-5].

The remainder of this *Introduction* demonstrates a profound connection between Fractal Physics Theory and E-infinity Theory.

E-infinity Theory is based on Fractal Geometry with the golden mean $0.5(\sqrt{5}-1) \sim 0.618034$ as one of its main pillars. Fractal Physics Theory is based on Fractal Geometry with its scaling fractals all in golden ratios, i.e., a cosmic scale object is to a self-similar quantum scale object as this quantum scale object is to a self-similar subquantum scale object (Subsection 2.1, Definitions, Scaling Fractal). The author has had success applying the golden ratio to explain the periodicity of atomic electron structure of the periodic chart [8]. The author also discovered the electron K-shell total energy average of the largest atoms is exactly $0.618034\,\mathrm{MeV}$ [9].

Fractal Physics Theory requires a function that extends to the infinitely large and approaches but never reaches the infinitely small (Section 7, Scaling Fractals and the

Fractal Dimension). The bijection formula meets these criteria (Table 1, Figure 1). The bijection formula's Menger-Urysohn dimensions call to mind Fractal Physics Theory's Scales of the Fractal Universe (Subsection 2.1, Definitions, Table 3, Figure 2).

Table 1. E-infinity bijection formula for some n values

Menger-Urysohn dimension n	Hausdorff dimension $d_c^{(n)}$
$339 + 2\varphi^3$	5.451211×10^{70}
339	4.343326×10^{70}
$274 + k/2$	1.181599×10^{57}
274	1.131425×10^{57}
114	4.126704×10^{23}
$114 - k$	3.783681×10^{23}
6	11.090170
5	6.854102
4	4.236068
3	2.618034
2	1.618034
1	1.000000
0	0.618034
–1	0.381966
–2	0.236068
–3	0.145898
–4	0.090170

$k/2 = 0.0901699,$ L. Hardy's probability for quantum entanglement [10].

$k = \varphi^3(1 - \varphi^3).$

$\varphi =$ golden mean $= (\sqrt{5} - 1)/2.$

A Euclidean hypercube visualizes the Hausdorff dimension $= 4 + \varphi^3 \approx 4.236068,$ which is intuitively embedded in dimension $= 5$ [3-5]. Fractal Physics Theory proposes the set of all an object's fractal dimensions be considered the "fifth" dimension, where objects and observers are incorporated into a coordinate system (Section 7, Scaling Fractals and the Fractal Dimension, Table 7, Figure 3).

Gerardo Iovane compares quantum values to astrophysical quantities while exploring scale invariant laws of a self-similar fractal universe [11]. Ji-Huan He depicts the hierarchy of a fractal Hilbert cube, connecting an atom to the solar system [12]. Fractal Physics similarly asserts a neutron midway through beta decay is fractally self-similar to our solar system (Section 3, Cosmic Scale Fractal Objects). Ji-Huan He attributes the Heisenberg Uncertainty principle to the hierarchical level location of the object being observed and the difference in the hierarchical levels of the object and the observer [12]. Fractal Physics Theory purports essentially the same idea with the fractal certainty principle, whereby the human scale can use subquantum scale photons (part of the energy of a neutrino) to observe quantum scale objects without altering the object observed (Subsection 11.3, Fractal certainty principle).

A great deal of the quantitative aspect of Fractal Physics Theory comes from identifying the pre-solar system mass as the mass of a cosmic scale neutron, and when viewed from the titanic scale, to be equivalent in every way to familiar neutrons observed from our human scale. Dividing these two "adjacent in scale hierarchy" neutron masses yields:

$$\text{Mass scaling fractal } (¥M) = 1.189533 \times 10^{57}.$$

The mass scaling fractal is used to determine the length scaling fractal $(¥L) = 3.788566 \times 10^{23}$.

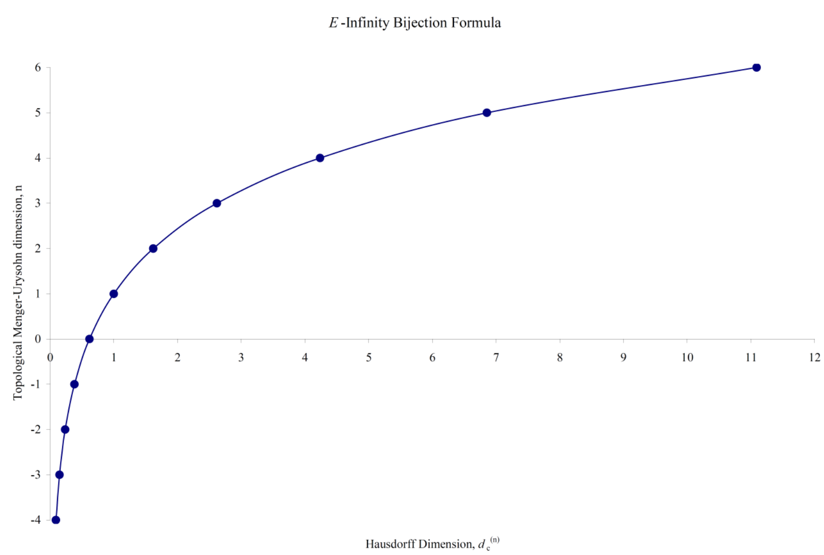

Figure 1. E-infinity bijection formula graph.

All the scaling fractals of Fractal Physics Theory have been obtained by combinations of the mass and length scaling fractals. The E-infinity transfinite corrections in Table 1 calculate Hausdorff dimensions very close to the Fractal Physics Theory phenomenological determined mass, length and volume scaling fractals (Section 7, Table 7).

The integer Menger-Urysohn dimension of the mass scaling fractal (274), the main pillar of Fractal Physics Theory, is obtained from E-infinity's bijection formula (Table 1). This dimension (274) divided by 2 equals the integral value of the inverse electromagnetic fine structure constant (137) calculated by E-infinity Theory.

$$274/2 = 137.$$

The integer Menger-Urysohn dimension of the mass scaling fractal (274) multiplied by 2 equals the sum of dimensions of all the exceptional Lie groups (548, Table 2) [6].

Table 2. Exceptional Lie group hierarchy

E_n	Dim E_n
8	248
7	133
6	78
5	45
4	24
3	11
2	6
1	3
Σ	548

$$274 \times 2 = 548.$$

The integer Menger-Urysohn dimension of the mass scaling fractal (274) equals the sum of the crisp dimension of E8 exceptional Lie Group (248, the most important symmetry group used in superstring theory) plus the number of dimensions in string theory (26) [3-5].

$$274 = 248 + 26.$$

The integer Menger-Urysohn dimension of the length scaling fractal (114) multiplied by 6 equals the dimension of Munroe's quasi exceptional Lie group $E_{12}(684)$ [13].

$$114 \times 6 = 684.$$

The integer Menger-Urysohn dimension of the volume scaling fractal (339) equals the 339 hierarchal degrees of freedom or isometries in the Beltrami-Poincaré representation of the compactified holographic boundary of E-infinity spacetime tiling the plane using Klein's-modular-curve [3-5].

This *Introduction* shall conclude with a profound link between FPT and a recent focus of the E-infinity group [10, 14]. A fractal point is both a point and an entire Cantor set on magnification. The quantum particle is represented by a Cantor point using the zero set having a two component dimension:

$$\text{dim (particle set)} = P(d_{\text{MU}}, d_{\text{H}}) = P(0, \varphi).$$

The quantum wave is represented by the emptiness between disjointed Cantorian points using the empty set having a two component dimension:

$$\text{dim (wave set)} = W(d_{\text{MU}}, d_{\text{H}}) = W(-1, \varphi^2),$$

where d_{MU} is the Menger-Urysohn dimension and d_{H} is the corresponding Cantorian or Hausdorff dimension.

A two dimension surface is the boundary of a three dimension cube. A one dimension line is the boundary of a two dimension surface. A zero dimension point is the boundary of a one dimension line. By inductive reasoning, a one dimension wave is the boundary of a zero dimension point. The quantum wave empty set is the surface boundary of the quantum particle zero set. E-infinity purports by this reasoning that the quantum wave is the surface of the quantum particle [10, 14].

Fractal Physics Theory purports a quantum particle is composed of an enormous number of subquantum scale atoms localized in solid phase - relative to the lilliputian scale, while a quantum wave is composed of these same subquantum scale atoms delocalized in gaseous and/or plasma phase - relative to the lilliputian scale.

2. Fractal Physics Theory - Definitions, Postulates and Scaling Fractals

2.1. Definitions

A curious observation provides a convenient way to define a set of scales. A familiar human scale mass $57.768\,\text{kg}$ divided by the quantum scale neutron mass $1.675 \times 10^{-27}\,\text{kg}$ equals 3.449×10^{28}. Multiplying $57.768\,\text{kg}$ by 3.449×10^{28} equals the cosmic scale solar system mass $1.992 \times 10^{30}\,\text{kg}$.

Object- any system of mass and/or energy that exists in the universe, symbolized by O. Examples of objects: electron, proton, neutron, atom, molecule, photon, person, car, house, planet, star, and galaxy.

Observable- the measurement of a property of an object, symbolized by $f(O)$. Examples of observables: an electron's charge, a person's height, a planet's mass, and a star's luminosity. For convenience, consider an observer always present to witness the observable.

Scale- a reference frame for establishing measurement units and defining their relationships.

Fractal Universe- the infinite set of scales, the scale contents, and the relationships between the scales, which comprise physical reality.

Human Scale- a reference frame scaled to the mass range of humans. The human observer is an object in a scale. The observer is part of their scale. The units meter, kilogram, and second are relative to a human's scale of size, mass, and awareness of time passage. Let an object located in the human scale be denoted by $m = 0$. Let an observation made in the human scale be denoted by $n = 0$.

Cosmic Scale- a reference frame scaled to the mass range of stars. Let an object located in the cosmic scale be denoted by $m = 1$. Let an observation made in the cosmic scale be denoted by $n = 1$.

Quantum Scale- a reference frame scaled to the mass range of atomic nuclei. Let an object located in the quantum scale be denoted $m = -1$. Let an observation made in the quantum scale be denoted by $n = -1$.

Titanic Scale- a reference frame more massive than but self-similar to the human scale. A reference frame scaled to the mass range of humans multiplied by the factor $¥M = (1.992 \times 10^{30}\,\text{kg})/(1.675 \times 10^{-27}\,\text{kg}) \sim 1.19 \times 10^{57}$. Let an object located in the titanic scale be denoted by $m = 2$. Let an observation made in the titanic scale be denoted by $n = 2$.

Lilliputian Scale- a reference frame less massive than but self-similar to the human scale. A reference frame scaled to the mass range of humans divided by the factor $¥M \sim 1.19 \times 10^{57}$. Let an object located in the lilliputian scale be denoted by $m = -2$. Let an observation made in the lilliputian scale be denoted by $n = -2$.

Supercosmic Scale- a reference frame more massive than but self-similar to the cosmic scale. A reference frame scaled to the mass range of stars multiplied by the factor $¥M \sim 1.19 \times 10^{57}$. Let an object located in the supercosmic scale be denoted by $m = 3$. Let an observation made in the supercosmic scale be denoted by $n = 3$.

Subquantum Scale- a reference frame less massive than but self-similar to the quantum scale. A reference frame scaled to the mass range of atomic nuclei divided by the factor $¥M \sim 1.19 \times 10^{57}$. Let an object located in the subquantum scale be denoted by $m = -3$. Let an observation made in the subquantum scale be denoted by $n = -3$.

Fractal object- extension of Fractal Geometry concepts of self-similarity and scale to an “Object”.

If an object exists at scale *m*, then a self-similar object could exist at any scale:

$$m_z = m \pm 2z, \tag{2}$$

where $z = (0, 1, 2, 3, ..., \infty)$.

For example, neutrons exist at the quantum scale $(m = -1)$; neutrons could exist at scales $m = -7, -5, -3, 1, 3, 5,$ etc.

Let an object and its observable be represented by:

$$[O, f(O)]_{m,n} \tag{3}$$

m is the object scale location,

n is the observable scale location,

m and *n* represent any scale as defined by the set $\{-\infty, ..., -2, -1, 0, 1, 2, ..., +\infty\}$.

For example, a quantum scale electron’s mass is represented by $[e-, \text{mass}]_{-1,0} = 9.11 \times 10^{-31}\text{kg}$.

A cosmic scale electron’s mass is represented by $[e-, \text{mass}]_{1,0} = 1.08 \times 10^{27}\text{kg}$.

A subquantum scale electron’s mass is represented by $[e-, \text{mass}]_{-3,0} = 7.66 \times 10^{-88}\text{kg}$.

Scale Relativity- the relativity of scale. The laws of physics are scale invariant. That is, the laws of physics are the same in every scale as viewed from that scale. The laws of physics in scale m as observed in scale $n = m$ are equivalent to the laws of physics in scale $m + x$ as observed in scale $n = m + x$, where m, n, and x are defined by the set $\{-\infty, ..., -2, -1, 0, 1, 2, ..., +\infty\}$. Every object exists within all the infinite scales simultaneously. Therefore, the measurement of any observable of an object requires the specification of two scales, the object's scale and the observer's scale. Every observer in any scale m, measures observables relative to their scale $n = m$, therefore every observer, in any scale may set $m = n = 0$. The term "Scalativity" is hereby introduced from a contraction of Scale Relativity.

To further generalize, all the observables of an object located at scale m, as measured from scale n, are equivalent to all the observables of a self-similar object located at scale $m + x$, as measured from scale $n + x$.

$$[O,\ f(O)]_{m,n} = [O,\ f(O)]_{m+x,n+x} \tag{4}$$

where m, n, and x are defined by the set $\{-\infty, ..., -2, -1, 0, 1, 2, ..., +\infty\}$.

For example, the subquantum electron's mass measured in the human scale ($[e-,\ \text{mass}]_{-3,0} = 7.66 \times 10^{-88}\,\text{kg}$) is equivalent to the electron's mass measured relative to the titanic scale ($[e-,\ \text{mass}]_{-1,2} = 7.66 \times 10^{-88}\,\text{kg}$).

Scaling Fractal- represented by the symbol ¥, is a unitless number that relates properties of self-similar objects through simple division. There are two ways to calculate a scaling fractal.

A. Two self-similar fractal objects located in scales separated by $\Delta m = +/-2$, have self-similar observables measured in the same scale (n). The scaling fractal is obtained by dividing these two measurements. For instance, a cosmic scale electron $(m = 1)$ has its mass measured in the human scale $(n = 0)$. An electron located in the quantum scale $(m = -1)$ has its mass measured in the human scale $(n = 0)$. The mass scaling fractal:

$$¥\text{Mass} = (\text{mass of cosmic scale electron})/(\text{mass of electron})$$

$$= ¥ f(O) = \frac{[O,\ f(O)]_{m+1,n}}{[O,\ f(O)]_{m-1,n}} \tag{5}$$

B. An object located in scale (m) has an observable measured relative to two scales separated by $\Delta n = +/-2$. The scaling fractal is obtained by dividing these two measurements. For instance, an electron located in the quantum scale $(m = -1)$ has its mass measured in the human scale $(n = 0)$ and also measured in the titanic scale $(n = 2)$. The mass scaling fractal:

¥Mass

$$= (\text{electron mass measured in human scale})/(\text{electron mass measured in titanic scale})$$

$$= ¥ f(O) = \frac{[O,\ f(O)]_{m-1,\,n}}{[O,\ f(O)]_{m-1,\,n+2}} \tag{6}$$

The value of the mass scaling fractal, ¥ Mass, from equations (5) and (6) must be equivalent:

$$\frac{[O,\ f(O)]_{m+1,\,n}}{[O,\ f(O)]_{m-1,\,n}} = \frac{[O,\ f(O)]_{m-1,\,n}}{[O,\ f(O)]_{m-1,\,n+2}} \tag{7}$$

Equation (7) is in the form of Geometry's celebrated *Divine Proportion*, Plato's Golden Mean - the division of a line into extreme and mean ratio.

Scaling Fractal Graph- plot(s) of scale number versus logarithm of scale unit (Table 3, Figure 2).

Fractal Dimension- relative slope of line on Scaling Fractal Graph (equations (17), (18)).

Table 3a. International system of units (SI) at several scales

Scale		Mass		Luminous Intensity		Length or Time	
m	Name	Unit (kg)	Log (unit)	Unit (cd)	Log (unit)	Unit (m or s)	Log (unit)
3	Supercosmic	4.10×10^{85}	85.61	1.76×10^{50}	50.25	2.33×10^{35}	35.37
2	Titanic	1.19×10^{57}	57.08	3.14×10^{33}	33.50	3.79×10^{23}	23.58
1	Cosmic	3.45×10^{28}	28.54	5.60×10^{16}	16.75	6.16×10^{11}	11.79
0	Human	1.00	0.00	1.00	0.00	1.00	0.00
–1	Quantum	2.90×10^{-29}	–28.54	1.78×10^{-17}	–16.75	1.62×10^{-12}	–11.79
–2	Lilliputian	8.41×10^{-58}	–57.08	3.18×10^{-34}	–33.50	2.64×10^{-24}	–23.58
–3	Subquantum	2.44×10^{-86}	–85.61	5.68×10^{-51}	–50.25	4.29×10^{-36}	–35.37
$\Delta\text{Log(unit)}/\Delta m =$		28.538		16.748		11.789	

Table 3b. International system of units (SI) at several scales

Scale		Current		Mole		Temperature	
m	Name	Unit (A)	Log(unit)	Unit (mol)	Log(unit)	Unit (K)	Log(unit)
3	Supercosmic	1.33×10^{25}	25.12	1.00	0.00	3.24×10^{-21}	–20.49
2	Titanic	5.60×10^{16}	16.75	1.00	0.00	2.19×10^{-14}	–13.66
1	Cosmic	2.37×10^{8}	8.37	1.00	0.00	1.48×10^{-7}	–6.83
0	Human	1.00	0.00	1.00	0.00	1.00	0.00
–1	Quantum	4.22×10^{-9}	–8.37	1.00	0.00	6.76×10^{6}	6.83
–2	Lilliputian	1.78×10^{-17}	–16.75	1.00	0.00	4.57×10^{13}	13.66
–3	Subquantum	7.54×10^{-26}	–25.12	1.00	0.00	3.09×10^{20}	20.49
	$\Delta\text{Log(unit)}/\Delta m =$	8.374		0.000		–6.830	

2.2. Postulates

I. *Scale Relativity*- absolute uniform scale cannot be detected.

Scientists (human observers) exist at the human scale whereby all measured properties of the Universe can be compared relative to the scientist's approximate mass, size, and awareness of time passage. FPT insists our human scale is arbitrary.

II. The speed (c) of electromagnetic radiation in vacuum is independent of scale.

Every observer in any scale obtains the same value for c. An electromagnetic radiation photon in vacuum located in a given scale m, has the same speed measured from any scale $n + x$:

$$[\text{photon, } c]_{m,n} = [\text{photon, } c]_{m,n+x} = 299792458\,\text{m/s}, \tag{8}$$

where m, n, and x are defined by the set $\{-\infty, ..., -2, -1, 0, 1, 2, ..., +\infty\}$.

Also, an observer in a given scale n, measures the same value for the speed in vacuum of electromagnetic radiation photons located in any scale $m + x$:

$$[\text{photon, } c]_{m,n} = [\text{photon, } c]_{m+x,n} = 299792458\,\text{m/s}, \tag{9}$$

where m, n, and x are defined by the set $\{-\infty, ..., -2, -1, 0, 1, 2, ..., +\infty\}$.

The photon's speed scaling fractal,

$$¥c = (299792458\,\text{m/s})/(299792458\,\text{m/s}) = 1. \tag{10}$$

Based on these two postulates, it is possible to reorganize the vast quantity of experimental data amassed by science into a much simpler, more symmetric, and more predictive framework. Many facets of Fractal Physics Theory already match experimental data, but more importantly, many experiments can be conducted that could refute FPT.

What are the fundamental fractal objects comprising our Fractal Universe? The vast majority of observations within the Solar System, the immediate neighborhood of the human scale, are described by the properties and interactions of quantum scale - atoms, photons, neutrinos, and their respective antiparticles. Identifying cosmic scale - atoms, photons, neutrinos, and their respective antiparticles is a major objective of Fractal Physics Theory.

2.3. Principle of scalativity

The Principle of Scalativity requires the laws of physics to be self-similar in every observer's scale. This allows a direct comparison of physical equations in different scales, leading to scaling fractal equations replacing the physical equations.

For example, $E = mc^2$ is written for objects at the human scale $(m = 0)$ as observed in the human scale $(n = 0)$ in the notation introduced in this article:

$$[E]_{0,0} = [M]_{0,0}[c]_{0,0}^2 \tag{11.a}$$

$E = mc^2$ is written for objects at the titanic scale $(m = 2)$ as observed in the human scale $(n = 0)$:

$$[E]_{2,0} = [M]_{2,0}[c]_{2,0}^2 \tag{11.b}$$

Divide equation (11.b) by equation (11.a) and substitute using equation (5):

$$¥\text{Energy} = (¥\text{Mass})(¥c)^2. \tag{11.c}$$

Using the identity from equation (10) in equation (11.c):

$$¥E = ¥M. \tag{11.d}$$

The energy scaling fractal equals the mass scaling fractal; energy and mass scale at the same rate.

Similarly, from $x = ct$, write

$$¥\text{length} = (¥c)(¥\text{time}) = (1)(¥\text{time}) = ¥\text{time}. \tag{12}$$

The length scaling fractal equals the time scaling fractal; length and time scale at the same rate.

Consequently, any object's velocity is scale invariant, $¥\text{velocity} = ¥\text{length}/¥\text{time} = 1$.

Lengths that are greatly dilated relative to the human scale correspond to greatly dilated times relative to the human scale. Cosmic scale time is dilated or decelerated relative to the human scale. Lengths that are greatly contracted relative to the human scale correspond to greatly contracted times relative to the human scale. Quantum scale time is contracted or accelerated relative to the human scale.

Figure 3 illustrates scales added to Cartesian Coordinates. Let the x, y, z coordinates be infinitely flexible. Imagine that the origin is being pulled inwards toward infinity. Measurement limits still exist for each scale when an observer located in the scale uses photons located in the same scale. Scale specific position uncertainties, Δx, Δy, Δz, are depicted in Figure 3 as coordinate line width, similar to Nottale [2].

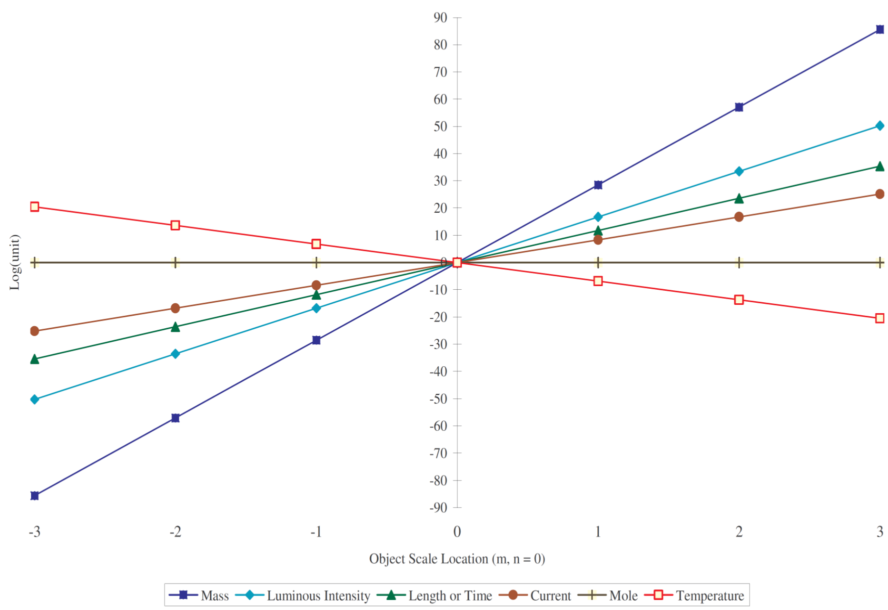

Figure 2. Scaling fractal graph of SI units.

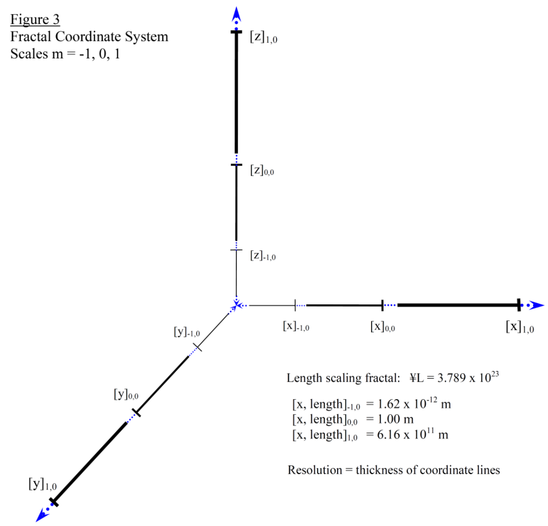

Figure 3. Fractal coordinate system scales $m = -1, 0, 1$.

3. Cosmic Scale Fractal Objects

To precede it is necessary to accurately identify the cosmic scale fractal objects. They will appear self-similar to protons, neutrons, nuclei, electrons, photons, neutrinos and their respective antiparticles; larger in size and mass but in a greatly decelerated time frame, relative to the human scale.

Modern Cosmology has amassed a staggering wealth of observational data and theoretical construct framed within the Big Bang Model. Fractal Physics Theory proposes this cosmic scale explosion occurs within a greatly slowed time frame, relative to the human scale. Readily visible astrophysical bodies include the stars, planets, galaxies, supernovae, etc.

Consider stars:

- have mass and radiate energy;
- undergo fusion which alters their chemical composition;
- have masses ranging over two orders of magnitude;
- exist individually, in binary and more complex systems;
- can explode with tremendous energy as nova and supernova;
- are found in galaxies.

Would an observer in the titanic scale be able to detect stars? Typically stars radiate photons, and to lesser extent neutrinos, spherically for billions of human scale years. Observers at the titanic scale would find it very difficult to construct an experiment that could detect these diffuse spherically radiated pulses of energy. Billions of human scale years are perceived as brief moments relative to the titanic scale.

What quantum scale objects, when observed for a very short time in the human scale, could resemble a tiny star?

Protons are stable masses that do not appear to radiate energy. Protons are not fractally self-similar to stars. Photons and neutrinos are essentially pure energy. Photons and neutrinos are not fractally self-similar to stars. Electrons all have the same rest mass but exist at variable energy levels in atoms. The variation in atomic electron mass-energy content is relatively small. Electrons are not fractally self-similar to stars. Neutron rest mass is constant, but when free neutrons undergo beta-decay there is an interesting change that occurs in the neutron in a very short time relative to the human scale.

$$n \rightarrow p^+ + e^- + \text{anti}\nu.$$

An electron is created or forged within the neutron while the neutron radiates antineutrino energy. The antineutrino has been notoriously difficult for observers to detect in the human scale. Nuclei that are in the process of the beta decay moment:

- have mass and radiate energy;
- undergo a change which alters their composition;
- have masses ranging over two orders of magnitude;
- exist individually, in binaries when capturing neutrons;
- can violently release beta particles with very high kinetic energy;
- are found in radioactive materials.

The vast majority of stars are fractally self-similar to nuclei in the process of beta decay. Stars are cosmic scale (cs) nuclei in the process of cs-beta decay as observed in the human scale. The luminous output of a star over its life, its sum of electromagnetic radiation and neutrino output, is one cosmic scale antineutrino. Stable cosmic scale nuclei and cosmic scale atomic electrons have surface temperatures of 2.7K. These are cold dark matter whose gravitational effects are observed in the human scale.

Only a nuclear explosion, about one second into the fireball, if viewed from the lilliputian scale, would have properties fractally self-similar to the human scale observations of a Big Bang with all galaxies hurdling away from each other. It is proposed that our Big Bang universe is the interior contents of one cosmic scale nuclear explosion occurring on a supercosmic scale planet into a titanic scale Oxygen atmosphere. Cosmic scale vaporized fission fragments and cs-unfissioned Uranium have already begun to condense, react with the cs-Oxygen and form crystals of cs-Uranium dioxide, the cores of spiral galaxies. Elliptical galaxies contain stars moving in random directions, the motion of which is fractally self-similar to particles suspended in a fluid. Cosmic scale vaporized fission fragments and cs-unfissioned Uranium have already begun to condense into ts-molten liquid drops and along with ts-water vapor droplets, capture cs-neutrons and cs-fission fragments to form elliptical galaxies.

Fission fragment products are typically centered around two mass peaks, A ~ 95 and A ~ 138. These fission fragments undergo a series of beta decays before reaching a stable nuclear endpoint. Consequently most stars are cosmic scale fission fragments in the process of cs-beta decay. This cosmic scale explosion will have many cs-free neutrons in flux colliding with cs-nuclei (suggested origin of gamma ray bursts), many cs-neutrons in the process of absorption by other nuclei (suggested origin of binary star systems), and a percentage of cs-neutrons undergoing cs-beta decay (suggested origin of the solar system). Cosmic scale nuclear explosion cosmology is further explored in a separate article.

The Heisenberg uncertainty principle is scale dependent. Use of antineutrino energy, which is mostly lilliputian scale electromagnetic radiation, can determine very accurately and simultaneously the position and momentum of an electron to the human scale. This prediction can be tested.

4. Quantum Camera

It is proposed that a quantum camera placed at various positions within a nuclear fission reactor can capture images, that when scaled-up, will resemble images of galaxies. All human scale astronomical pictures imaged with electromagnetic radiation have been imaged using bits of cosmic scale antineutrino energy. Consequently, observers at the lilliputian scale could utilize subquantum scale (sqs) electromagnetic radiation to image their visible qs-universe (Tables 4a, 4b). Can a quantum camera be constructed to image sqs-electromagnetic radiation (antineutrino energy)?

Table 4a. Exposure times to image antineutrino energy

Exposure time in human scale $(m = 0)$ As measured in human scale $(n = 0)$	Exposure time in human scale $(m = 0)$ As measured in lilliputian scale $(n = -2)$
$0.83\times10^{-6}\text{s}$	10000000000 y
$1\times10^{-9}\text{s}$	12000000 y
$83\times10^{-12}\text{s}$	1000000 y
$1\times10^{-12}\text{s}$	12000 y
$1\times10^{-15}\text{s}$	12 y
$1\times10^{-18}\text{s}$	4.4 d

Table 4b. Aperture sizes to image antineutrino energy

Aperture size in human scale $(m = 0)$ As measured in human scale $(n = 0)$	Aperture size in human scale $(m = 0)$ As measured in lilliputian scale $(n = -2)$
$1\times10^{-10}\text{m}$	253.25 AU
$1\times10^{-12}\text{m}$	2.53 AU
$1\times10^{-15}\text{m}$	$379000\,\text{km} = 0.5\,\text{Solar radii}$
$1\times10^{-18}\text{m}$	379 km

The closer the exposure time gets to the attosecond and the aperture size gets to the attometer the more the image resolution will resemble human scale astronomical images. Antineutrino energy gathered with femtosecond exposures through femtometer apertures should capture images of qs-galaxies, provided a suitable recording material is found. The quantum camera must collect and channel the sqs-photons for amplification. The sqs-photon amplification process must continue until it is capable of affecting a photon.

5. Mass Scaling Fractal, ¥Mass

Identifying the pre-solar system mass with the mass of a cosmic scale neutron is all that is required to establish values for the mass scaling fractal and the length scaling fractal, ¥Mass, ¥Length. These two scaling fractals and the Principle of Scalativity is all that is required to derive the scaling fractals for a multitude of scientific variables and constants. The mass scaling fractal will be determined by dividing the mass of a cosmic scale neutron by the mass of a quantum scale neutron, both measured in the human scale. The qs-neutron mass is well established as $1.67492728\times10^{-27}\,\text{kg}$.

The solar system is proposed to be a cosmic scale neutron about halfway through the process of cosmic scale beta decay. It is assumed that all the mass of all the objects in the solar system were once contained in one massive, frozen body of mostly hydrogen, a cosmic scale neutron. All the electromagnetic radiation radiated by the Sun during the past 5×10^9 y needs to have its mass equivalent included in the mass of a cosmic scale neutron (Tables 5a, 5b).

Table 5a. Solar system planetary masses [15]

Object	Mass (kg)	AU
Sun	1.9891×10^{30}	
Mercury	3.3022×10^{23}	0.38710
Venus	4.8690×10^{24}	0.72333
Earth	5.9742×10^{24}	1.00000
Mars	6.4191×10^{23}	1.52369
Jupiter	1.8988×10^{27}	5.20283
Saturn	5.685×10^{26}	9.53876
Uranus	8.6625×10^{25}	19.19139
Neptune	1.0278×10^{26}	30.06107
Pluto	1.5×10^{22}	39.52940
$\Sigma_{planets}$	$2.668\,535 \times 10^{27}$	

Table 5b. Solar system satellite masses [15]

Planet	Planet Mass (kg)	Satellite	Satellite relative mass	Satellite Mass (kg)
Earth	5.9742×10^{24}	Moon	0.01230002	7.3483×10^{22}
Mars	6.4191×10^{23}	Phobos	1.5×10^{-08}	9.6287×10^{15}
		Deimos	3×10^{-09}	1.9257×10^{15}
Jupiter	1.8988×10^{27}	Io	4.68×10^{-05}	8.8864×10^{22}
		Europa	2.52×10^{-05}	4.7850×10^{22}
		Ganymede	7.80×10^{-05}	1.4811×10^{23}
		Callisto	5.66×10^{-05}	1.0747×10^{23}
		Amalthea	3.8×10^{-09}	7.2154×10^{18}
		Himalia	5.0×10^{-09}	9.4940×10^{18}
		Elara	4×10^{-10}	7.5952×10^{17}
		Pasiphae	1×10^{-10}	1.8988×10^{17}

		Sinope	4×10^{-11}	7.5952×10^{16}
		Lysithea	4×10^{-11}	7.5952×10^{16}
		Carme	5×10^{-11}	9.4940×10^{16}
		Ananke	2×10^{-11}	3.7976×10^{16}
		Leda	3×10^{-12}	5.6964×10^{15}
		Thebe	4×10^{-10}	7.5952×10^{17}
		Adrastea	1×10^{-11}	1.8988×10^{16}
		Metis	5×10^{-11}	9.4940×10^{16}
Saturn	5.685×10^{26}	Mimas	8.0×10^{-08}	4.5480×10^{19}
		Enceladus	1.3×10^{-07}	7.3905×10^{19}
		Tethys	$1.3\times10^{-0.6}$	7.3905×10^{20}
		Dione	1.85×10^{-06}	1.0517×10^{21}
		Rhea	4.4×10^{-06}	2.5014×10^{21}
		Titan	2.38×10^{-04}	1.3530×10^{23}
		Hyperion	3×10^{-08}	1.7055×10^{19}
		Iapetus	3.3×10^{-06}	1.8761×10^{21}
		Phoebe	7×10^{-10}	3.9795×10^{17}
Uranus	8.6625×10^{25}	Ariel	1.56×10^{-05}	1.3514×10^{21}
		Umbriel	1.35×10^{-05}	1.1694×10^{21}
		Titania	4.06×10^{-05}	3.5170×10^{21}
		Oberon	3.47×10^{-05}	3.0059×10^{21}
		Miranda	8×10^{-07}	6.9300×10^{19}
Neptune	1.0278×10^{26}	Triton	2.09×10^{-04}	2.1481×10^{22}
		Nereid	2×10^{-07}	2.0556×10^{19}
Pluto	1.5×10^{22}	Charon	0.22	3.3×10^{21}

Total mass of solar system planetary satellites: 6.413163×10^{23} kg.

A solar luminosity of 3.8418×10^{26} W radiating for 4.54×10^{9} y releases 5.504204641×10^{43} J, which has a mass equivalent of 6.124253602×10^{26} kg.

Current mass of sun:	1.9891×10^{30} kg
Total mass of planets:	2.668535×10^{27} kg
Total mass of planetary satellites:	6.413163×10^{23} kg

Mass loss from Sun shining 4.54×10^9 y:	6.124253602×10^{26} kg
Total solar system mass 4.54×10^9 y ago:	1.992381602×10^{30} kg

Therefore the cosmic scale neutron mass is estimated here to be 1.992381602×10^{30} kg.

Mass Scaling Fractal,

$$¥\text{Mass} = (\text{cosmic scale neutron mass})/(\text{quantum scale neutron mass}),$$

$$¥\text{Mass} = (1.992381602\times10^{30}\,\text{kg})/(1.67492728\times10^{-27}\,\text{kg})$$

$$= 1.189533\times10^{57}. \tag{13}$$

6. Cosmic Scale Proton Radius Estimate and the Length Scaling Fractal

6.1. Cosmic scale proton radius estimate

$$\text{Cosmic scale proton mass} = (\text{proton's mass})(1.189533\times10^{57})$$

$$= 1.989639\times10^{30}\,\text{kg}.$$

If one starts with the mass of a cs-neutron composed of 100% H_2 molecules and allows these H_2 molecules to fuse a mass of iron equal to the mass of a cs-electron, then to obtain the calculated cs-proton mass, the following composition is necessary: 88.635652% H_2 and 11.364348% He. Nucleon composition is more extensively explored in the fourth article of this series. The size of the cs-proton is estimated from the atomic densities of Hydrogen and Helium.

Table 6. Hydrogen atomic radii [15]

	Internuclear distance	Atomic radius	Abundance (%)
$^1H_2$	0.74144 Å	0.37072 Å	99.9885
$^2H_2$	0.74152 Å	0.37076 Å	0.0115
$^3H_2$	0.74142 Å	0.37071 Å	

The Hydrogen atomic volume in the cs-proton is determined using $1/2$ the inter-nuclear distance of the hydrogen molecule (Table 6). A Hydrogen radius 3.7072×10^{-11}m yields a Hydrogen atomic volume $2.134158442\times10^{-31}\text{m}^3$.

$$\text{Hydrogen atom mass} = (1.007940754\ \text{amu})(1.66053886\times10^{-27}\ \text{kg/amu})$$

$= 1.673724791 \times 10^{-27}$ kg.

Hydrogen atom density = $(1.673724791 \times 10^{-27}\,\text{kg})/(2.134158442 \times 10^{-31}\,\text{m}^3)$

$= 7842.551697\,\text{kg/m}^3$.

Helium atom radius = 0.2734 Å, results in a Helium atom volume

$= 8.559562042 \times 10^{-32}\,\text{m}^3$.

Helium atom mass = $(4.002601932\,\text{amu})(1.66053886 \times 10^{-27}\,\text{kg/amu})$

$= 6.646476049 \times 10^{-27}$ kg.

Helium atom density = $(6.646476049 \times 10^{-27}\,\text{kg})/(8.559562042 \times 10^{-32}\,\text{m}^3)$

$= 77649.721053\,\text{kg/m}^3$.

M_T = cs-proton mass : $1.989639050 \times 10^{30}$ kg.

d_1 = Hydrogen atomic density : $7842.551697\,\text{kg/m}^3$.

d_2 = Helium atomic density : $77649.721053\,\text{kg/m}^3$.

m_1 = Hydrogen mass = $(0.88635652)(1.989639050 \times 10^{30}\,\text{kg})$

$= 1.763529544 \times 10^{30}$ kg.

m_2 = Helium mass = $(0.11364348)(1.989639050 \times 10^{30}\,\text{kg})$

$= 2.261095056 \times 10^{29}$ kg).

v_1 = Hydrogen volume = $(1.763529544 \times 10^{30}\,\text{kg})/(7842.551697\,\text{kg/m}^3)$

$= 2.248668051 \times 10^{26}\,\text{m}^3$.

v_2 = Helium volume = $(2.261095056 \times 10^{29}\,\text{kg})/(77649.721053\,\text{kg/m}^3)$

$= 2.911916521 \times 10^{24}\,\text{m}^3$.

$V_T = 2.248668051 \times 10^{26}\,\text{m}^3 + 2.911916521 \times 10^{24}\,\text{m}^3 = 2.277787216 \times 10^{26}\,\text{m}^3$.

$D_T = (1.989639050 \times 10^{30}\,\text{kg})/(2.277787216 \times 10^{26}\,\text{m}^3) = 8734.964513\,\text{kg/m}^3$.

Cosmic scale proton radius, $R_T = 3.788565912 \times 10^{8}\,\text{m} = 378856591.2\,\text{m}$.

6.2. Length scaling fractal, ¥Length

Experiments reveal the densities of nuclei are fairly constant, although there appears to be a density decrease towards nuclear surfaces. The cs-proton is used to calculate the length scaling fractal. A nuclear radius formula is:

$$R = R_0 A^{1/3}, \tag{14}$$

where $R_0 = 1.0$ to $1.5\,\text{fm}$, A = nuclear mass number = 1 for the proton or neutron, Minimum proton radius = 1.0 fm.

Length Scaling Fractal = (cosmic scale proton radius)/(quantum scale proton radius),

Length Scaling Fractal, ¥Length = $(3.788565912 \times 10^8\,\text{m})/(1.0\,\text{fm})$

$$= 3.788566 \times 10^{23}. \tag{15}$$

7. Scaling Fractals and the Fractal Dimension

A function that extends to the infinitely large and approaches but never reaches the infinitely small is sought. Logarithmic functions such as $y = \ln(x)$, or equivalently $x = e^y$, are good candidates as is the fractal function $p = q^D$, where D is the fractal self-similarity dimension, p is the number of self-similar pieces, and q is the scaling reduction factor applied to obtain the pieces. Scaling fractals considered thus far have been obtained from the Principle of Scalativity and are reducible to the product of the length scaling fractal and the mass scaling fractal each raised to their respective quantized numbers j, k.

$$¥f(O) = ¥L^{j/2}¥M^{k/2},$$
$$j, k = \{-\infty, \ldots, -2, -1, 0, 1, 2, \ldots, +\infty\}. \tag{16}$$

Values of $¥f(O)$ are listed in Table 7 to the sixth decimal place only to aid in finding scaling relationships. The scaling fractal value certainties derive from the certainty of the mass and length scaling fractals. The mass scaling fractal may be accurate to 3 significant figures, while the length scaling fractal is even less certain. From the fractal dimension function $p = q^D$, a similar scaling fractal function is written:

$$¥f(O) = ¥L^F,$$

$$¥L = \text{length scaling fractal} = 3.788565912 \times 10^{23},$$

$$F = \text{Fractal Dimension of an observable, } F = \ln ¥f(O)/\ln ¥L. \tag{17}$$

Equating equations (16) and (17): $¥L^{j/2}¥M^{k/2} = ¥L^F$

provides: $F = j/2 + 0.5k[\ln ¥M/\ln ¥L] = 0.5j + 1.210328\,k.$ (18)

Physical fractal dimensions can be positive or negative and are not necessarily integers. Consider the fifth dimension to be the set of all an object's fractal dimensions (Figure 4). The fifth dimension incorporates objects and observers into the coordinate system. The SI unit scaling fractals are listed in Table 8.

Table 7. Scaling fractals and fractal dimensions

Y$f(O)$	Observable	Y$L^{j/2}$Y$M^{k/2}$	j	k	Scaling Fractal	F
¥L	length	¥L	2	0	3.788566×10^{23}	1.0000
¥t	time	¥L	2	0	3.788566×10^{23}	1.0000
¥λ	wavelength	¥L	2	0	3.788566×10^{23}	1.0000
¥M	mass	¥M	0	2	1.189533×10^{57}	2.4207
¥E	energy	¥M	0	2	1.189533×10^{57}	2.4207
¥f	frequency	¥L^{-1}	2	0	2.639521×10^{-24}	1.0000
¥**a**	linear acceleration	¥L^{-1}	2	0	2.639521×10^{-24}	1.0000
¥**p**	linear momentum	¥M	0	2	1.189533×10^{57}	2.4207
YVol	volume	YL^3	6	0	5.437816×10^{70}	3.0000
YA	area	YL^2	4	0	1.435323×10^{47}	2.0000
¥D	density	¥L^{-3}¥M	6	2	2.187520×10^{-14}	0.5793
¥Q	charge	¥$L^{1/2}$¥$M^{1/2}$	1	1	2.122881×10^{40}	1.7103
¥Q/M	charge per mass	¥$L^{1/2}$¥$M^{-1/2}$	1	1	1.784634×10^{-17}	0.7103
¥M/Q	mass per charge	¥$L^{-1/2}$¥$M^{1/2}$	– 1	1	5.603390×10^{16}	0.7103
YSA/Vol	surface area to volume ratio	YL^{-1}	– 2	0	2.639521×10^{-24}	– 1.0000
YQ/SA	electric flux density	Y$L^{-3/2}$Y$M^{1/2}$	– 3	1	1.479027×10^{-7}	– 0.2897
¥h	Planck's constant	¥L¥M	2	2	4.506625×10^{80}	3.4207
¥S	spin	¥L¥M	2	2	4.506625×10^{80}	3.4207
¥**L**	angular momentum	¥L¥M	2	2	4.506625×10^{80}	3.4207
¥$\boldsymbol{\mu}$	magnetic dipole moment	¥$L^{3/2}$¥$M^{1/2}$	3	1	8.042676×10^{63}	2.7103
¥EDM	electric dipole moment	¥$L^{3/2}$¥$M^{1/2}$	3	1	8.042676×10^{63}	2.7103
¥c	speed of light	¥L^0¥M^0	0	0	1	0.0000
¥v	velocity	¥L^0¥M^0	0	0	1	0.0000
¥k_C	coulomb's constant	¥L^0¥M^0	0	0	1	0.0000
¥ε_0	permittivity	¥L^0¥M^0	0	0	1	0.0000
¥μ_0	permeability	¥L^0¥M^0	0	0	1	0.0000
¥×	cross product	¥L^0¥M^0	0	0	1	0.0000
¥Ω	Ohm, resistance	¥L^0¥M^0	0	0	1	0.0000
¥G	gravitational constant	¥L¥M^{-1}	2	– 2	3.184918×10^{-34}	– 1.4207
¥**g**	gravitational field	¥L^{-1}	– 2	0	2.639521×10^{-24}	– 1.0000
¥**E**	electric field	¥$L^{-3/2}$¥$M^{1/2}$	– 3	1	1.479027×10^{-7}	– 0.2897
¥**B**	magnetic field	¥$L^{-3/2}$¥$M^{1/2}$	3	1	1.479027×10^{-7}	0.2897
¥$\mathbf{F}_g$	gravitational force	¥L^{-1}¥M	2	2	3.139798×10^{33}	1.4207

¥$\mathbf{F_E}$	electric force	¥L^{-1}¥M	– 2	2	3.139798×10^{33}	1.4207
¥$\mathbf{F_M}$	magnetic force	¥L^{-1}¥M	– 2	2	3.139798×10^{33}	1.4207
¥i	current	¥$L^{-1/2}$¥$M^{1/2}$	– 1	1	5.603390×10^{16}	0.7103
¥Volt	voltage	¥$L^{-1/2}$¥$M^{1/2}$	– 1	1	5.603390×10^{16}	0.7103
¥N_A	Avogadro's #	¥L^0¥M^0	0	0	1	0.0000
¥mol	grams/molecular weight	¥L^0¥M^0	0	0	1	0.0000
¥C_Q	specific heat capacity	¥L^3¥M^{-1}	6	– 2	4.571387×10^{13}	0.5793
¥TC	thermal conductivity	¥L	2	0	3.788566×10^{23}	1.0000
¥P	power	¥L^{-1}¥M	– 2	2	3.139798×10^{33}	1.4207
¥T	temperature	¥L^{-3}¥M	– 6	2	2.187520×10^{-14}	– 0.5793
¥Pr	pressure	¥L^{-3}¥M	– 6	2	2.187520×10^{-14}	– 0.5793
¥k	Boltzmann constant	¥L^3	6	0	5.437816×10^{70}	3.0000
¥R	gas constant	¥L^3	6	0	5.437816×10^{70}	3.0000
¥σ	Stefan-Boltzmann const	¥L^9¥M^{-3}	18	– 6	9.553091×10^{40}	1.7380
¥Wie	Wien's const	¥L^{-2}¥M	– 4	2	8.287564×10^{9}	0.4207
¥α	polarizability	¥L^3	6	0	5.437816×10^{70}	3.0000
¥Cap	capacitance, F = C/Volt	¥L	2	0	3.788566×10^{23}	1.0000
¥cd	luminous intensity, cd	¥L^{-1}¥M	– 2	2	3.139798×10^{33}	1.4207
¥Lum	luminance, cd/m^2	¥L^{-3}¥M	– 6	2	2.187520×10^{-14}	– 0.5793
¥Ent	entropy	¥L^3	6	0	5.437816×10^{70}	3.0000
¥sr	steradian, solid angle	¥L^0¥M^0	0	0	1	0.0000
¥rad	radian, plane angle	¥L^0¥M^0	0	0	1	0.0000
¥$\boldsymbol{\omega}$	angular velocity	¥L^{-1}	– 2	0	2.639521×10^{-24}	– 1.0000
¥$d\omega/dt$	angular acceleration	¥L^{-2}	– 4	0	6.967072×10^{-48}	– 2.0000
¥I	moment of inertia	¥L^2¥M	4	2	1.707365×10^{104}	4.4207
¥Φ_0	magnetic flux quantum	¥$L^{1/2}$¥$M^{1/2}$	1	1	2.122881×10^{40}	1.7103
¥wb	magnetic flux, weber	¥$L^{1/2}$¥$M^{1/2}$	1	1	2.122881×10^{40}	1.7103
¥H	inductance, Henry	¥L	2	0	3.788566×10^{23}	1.0000
¥cir	quantum of circulation	¥L	2	0	3.788566×10^{23}	1.0000
¥ST	surface tension, N/m	¥L^{-2}¥M	– 4	2	8.287564×10^{9}	0.4207
¥Vis	viscosity, Pa•s	¥L^{-2}¥M	– 4	2	8.287564×10^{9}	0.4207
¥Flu	fluidity, m•s/kg	¥L^2¥M^{-1}	4	– 2	1.206627×10^{-10}	– 0.4207
¥Q/Vol	charge density	¥$L^{-5/2}$¥$M^{1/2}$	– 5	1	3.903922×10^{-31}	– 1.2897
¥Ψ	wave function	¥$L^{-3/2}$	– 3	0	4.288325×10^{-36}	– 1.5000
¥Ψ^2	probability density	¥L^{-3}	– 6	0	1.838973×10^{-71}	– 3.0000
¥V^2/K^2	Lorentz coefficient	¥L^5¥M^{-1}	10	– 2	6.561417×10^{60}	2.5793
¥DS	density of states, 1/J	¥M^{-1}	0	– 2	8.406659×10^{-58}	– 2.4207
¥K^{-1}	linear expansion coefficient	¥L^3¥M^{-1}	6	– 2	4.571387×10^{13}	0.5793
¥K/Pa	Joule-Thomson coefficient	¥L^0¥M^0	0	0	1	0.0000
¥Col	collision #, $m^{-3}s^{-1}$	¥L^{-4}	– 8	0	4.854009×10^{-95}	– 4.0000
¥BBR	black body radiation	¥L^{-4}¥M	– 8	2	5.774005×10^{-38}	– 1.5793
¥E¥B¥g	natural force fields	¥L^{-4}¥M	– 8	2	5.774005×10^{-38}	– 1.5793
¥Vol-¥t	space-time	¥L^4	8	0	2.060153×10^{94}	4.0000

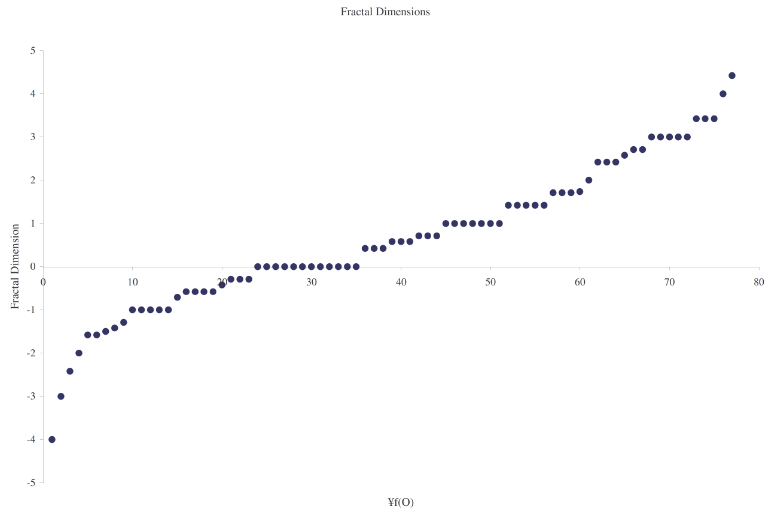

Figure 4. The fifth dimension (plotted from Table 7 fractal dimensions).

Table 8. International system of units (SI), scaling fractals and fractal dimensions

Base quantity	Name	Unit	¥*f*(*O*)	Scaling Fractal	*F*
Length	Meter	m	¥*L*	3.788566×10^{23}	1.0000
Mass	Kilogram	kg	¥*M*	1.189534×10^{57}	2.4207
Time	Second	s	¥*t*	3.778566×10^{23}	1.0000
Electric current	Ampere	A	¥*i*	5.603390×10^{16}	0.7103
Thermodynamic temperature	Kelvin	K	¥*T*	2.187520×10^{-14}	– 0.5793
Amount of substance	Mole	mol	¥mol	1	0.0000
Luminous intensity	Candela	cd	¥cd	3.139798×10^{33}	1.4207

8. Constants of Three Self-similar Scales Measured Relative to Human Scale Units

8.1. Human scale constants measured in human scale units (Table 9)

Table 9. Human scale constants in human scale units [15]

c	299792458	m/s	Speed of emr in vacuum
ε_0	$8.854187817 \times 10^{-12}$	$C^2/(Nm^2)$	Exact - permittivity or electric constant
μ_0	$4\pi \times 10^{-7}$	$N/(Amp)^2$	Exact - permeability or magnetic constant
k_C	8987551788	Nm^2/C^2	Coulomb constant, $1/(4\pi\varepsilon_0)$
$[\text{e, charge}]_{-1,0}$	$1.60217653 \times 10^{-19}$	C	Electron charge
$[\text{e, mass}]_{-1,0}$	$9.1093826 \times 10^{-31}$	kg	Electron mass
$[\text{p, mass}]_{-1,0}$	$1.67262171 \times 10^{-27}$	kg	Proton mass
$[\text{n, mass}]_{-1,0}$	$1.67492728 \times 10^{-27}$	kg	Neutron mass
$[k]_{-1,0}$	$1.3806505 \times 10^{-23}$	J/K	Boltzmann constant, R/N_A
$[h]_{-1,0}$	$6.6260693 \times 10^{-34}$	Js	Planck constant
$[G]_{1,0}$	6.6742×10^{-11}	$m^3/(s^2kg)$	Gravitational constant
N_A	6.0221415×10^{23}	1/mol	Avogadro constant
$[R]_{-1,0}$	8.314472	J/(molK)	Molar gas constant

$[F]_{-1,0}$	96485.3383	C/mol	Faraday constant, $N_A e$
$[\sigma]_{-1,0}$	5.670400×10^{-8}	$W/(m^2K^4)$	Stefan-Boltzmann constant
$[eV]_{-1,0}$	$1.60217653 \times 10^{-19}$	J	Electron volt
$[u]_{-1,0}$	$1.66053886 \times 10^{-27}$	kg	Atomic mass unit, 931.494047 MeV
$[\mathbf{\mu_B}]_{-1,0}$	$9.27400949 \times 10^{-24}$	J/T	Bohr magneton, $0.5e\hbar/m_{e-}$
$[\mathbf{\mu_N}]_{-1,0}$	$5.05078343 \times 10^{-27}$	J/T	Nuclear magneton, $0.5e\hbar/m_p$
$[a_0]_{-1,0}$	$5.291772108 \times 10^{-11}$	m	Bohr radius, $4\pi\varepsilon_0\hbar^2/(m_{e-}e^2)$
v	$2.187691263 \times 10^{6}$	m/s	H-atom n = 1, electron orbital velocity

8.2. Lilliputian scale constants measured in human scale units

Lilliputian scale constants measured in human scale units calculated using Tables 7, 9 (see Table 10).

Table 10. Lilliputian scale constants in human scale units

c	299792458	m/s	Speed of emr in vacuum
ε_0	$8.854187817 \times 10^{-12}$	$C^2/(Nm^2)$	Electric constant
μ_0	$4\pi \times 10^{-7}$	H/m	Exact – permeability constant
k_C	8987551788	Nm^2/C^2	Coulomb constant, $1/(4\pi\varepsilon_0)$
$[e, charge]_{-3,0}$	$7.547179141 \times 10^{-60}$	C	Electron charge
$[e, mass]_{-3,0}$	$7.657947355 \times 10^{-88}$	kg	Electron mass
$[p, mass]_{-3,0}$	$1.406116041 \times 10^{-84}$	kg	Proton mass
$[n, mass]_{-3,0}$	$1.408054255 \times 10^{-84}$	kg	Neutron mass
$[k]_{-3,0}$	$2.538979606 \times 10^{-94}$	J/K	Boltzmann constant
$[h]_{-3,0}$	$1.470295269 \times 10^{-114}$	Js	Planck constant
$[G]_{-1,0}$	$2.095564033 \times 10^{23}$	$m^3/(s^2kg)$	Gravitational constant
N_A	$6.02214150 \times 10^{23}$	1/mol	Avogadro constant
$[R]_{-3,0}$	$1.529009322 \times 10^{-70}$	J/(molK)	Molar gas constant
$[F]_{-3,0}$	$4.545018099 \times 10^{-36}$	C/mol	Faraday constant
$[\sigma]_{-3,0}$	$5.935670299 \times 10^{-49}$	$W/(m^2K^4)$	Stefan-Boltzmann constant
$[eV]_{-3,0}$	$1.346895180 \times 10^{-76}$	J	Electron volt
$[u]_{-3,0}$	$1.395958401 \times 10^{-84}$	kg	amu
$[\mathbf{\mu_B}]_{-3,0}$	$1.153100025 \times 10^{-87}$	J/T	Bohr magneton
$[\mathbf{\mu_N}]_{-3,0}$	$6.279979017 \times 10^{-91}$	J/T	Nuclear magneton
$[a_0]_{-3,0}$	$1.396774461 \times 10^{-34}$	m	Bohr radius
v	$2.187691263 \times 10^{6}$	m/s	H-atom n = 1, electron orbital velocity

8.3. Titanic scale constants measured in human scale units

Titanic scale constants measured in human scale units calculated using Tables 7, 9 (see Table 11).

Table 11. Titanic scale constants in human scale units

c	299792458	m/s	Speed of emr in vacuum
ε_0	$8.854187817 \times 10^{-12}$	F/m	Exact - permittivity constant
μ_0	$1.25663706143 \times 10^{-6}$	H/m	Exact - permeability constant
k_C	898 7551788	Nm^2/C^2	Coulomb constant, $1/(4\pi\varepsilon_0)$
$[e, charge]_{1,0}$	$3.401230560 \times 10^{21}$	C	Electron charge
$[e, mass]_{1,0}$	$1.083591301 \times 10^{27}$	kg	Electron mass
$[p, mass]_{1,0}$	$1.989639050 \times 10^{30}$	kg	Proton mass
$[n, mass]_{1,0}$	$1.992381602 \times 10^{30}$	kg	Neutron mass

$[k]_{1,0}$	$7.507723963 \times 10^{47}$	J/K	Boltzmann constant
$[h]_{1,0}$	$2.986120904 \times 10^{47}$	Js	Planck constant
$[G]_{3,0}$	$2.125678096 \times 10^{-44}$	$m^3/(s^2kg)$	Gravitational constant
N_A	$6.022141500 \times 10^{23}$	1/mol	Avogadro constant
$[R]_{1,0}$	$4.521257239 \times 10^{71}$	J/(molK)	Molar gas constant
$[F]_{1,0}$	$2.048269183 \times 10^{45}$	C/mol	Faraday constant
$[\sigma]_{1,0}$	$5.416984862 \times 10^{33}$	$W/(m^2K^4)$	Stefan-Boltzmann constant
$[\mathrm{eV}]_{1,0}$	$1.905842170 \times 10^{38}$	J	Electron volt
$[u]_{1,0}$	$1.975266099 \times 10^{30}$	kg	amu
$[\boldsymbol{\mu}_\mathbf{B}]_{1,0}$	$7.458785027 \times 10^{40}$	J/T	Bohr magneton
$[\boldsymbol{\mu}_\mathbf{N}]_{1,0}$	$4.062181289 \times 10^{37}$	J/T	Nuclear magneton
$[a_0]_{1,0}$	$2.004822742 \times 10^{13}$	m	Bohr radius = 134 AU
v	$2.187691263 \times 10^{6}$	m/s	H-atom n = 1, electron orbital velocity

If lilliputian scale constants are presented in lilliputian scale units as in Table 10 and titanic scale constants are presented in titanic scale units as in Table 11, then Tables 9, 10, and 11 would have identical values. For example, the Boltzmann constant would be:

$$[k]_{-1,0} = [k]_{-3,-2} = [k]_{1,2} = 1.3806505 \times 10^{-23} \text{ J/K}.$$

9. Fundamental Fractal Objects

Observables for electrons, protons, and neutrons located in two scales as measured in the human scale.

9.1. Quantum scale electron properties relative to the human scale

$[e^-$, radius$]_{-1,0}$	$7.231425325 \times 10^{-17}$	m	
$[e^-$, mass$]_{-1,0}$	$9.1093826 \times 10^{-31}$	kg	
$[e^-$, charge$]_{-1,0}$	$-1.60217653 \times 10^{-19}$	C	
$[e^-$, spin $(\mathbf{S})]_{-1,0}$	$9.132858671 \times 10^{-35}$	Js	$= 0.5\hbar\sqrt{3}$
$[e^-$, spin $(\mathbf{S_Z})]_{-1,0}$	$5.272858412 \times 10^{-35}$	Js	$= 0.5\hbar$
$[e^-$, surface area$]_{-1,0}$	$6.571396554 \times 10^{-32}$	m^2	
$[e^-$, volume$]_{-1,0}$	$1.584018782 \times 10^{-48}$	m^3	
$[e^-$, density$]_{-1,0}$	$5.750804664 \times 10^{17}$	kg/m^3	
$[e^-$, $q/M]_{-1,0}$	$-1.758820109 \times 10^{11}$	C/kg	
$[e^-$, $\mu_S]_{-1,0}$	$-9.284764116 \times 10^{-24}$	J/T	$= -1.0011596522\ \mu_B$
$[e^-$, q/surface area$]_{-1,0}$	$-2.438106599 \times 10^{12}$	C/m^2	
$[e^-$, composition$]_{-1,0}$	excess of 2.123×10^{40} [electrons]$_{-3,0}$ on magnetic sphere of 1.169×10^{52} [Iron atoms]$_{-3,0}$		

9.2. Quantum scale proton properties relative to the human scale

$[p^+$, radius$]_{-1,0}$	1.000000000	fm	
$[p^+$, mass$]_{-1,0}$	$1.67262171 \times 10^{-27}$	kg	
$[p^+$, charge$]_{-1,0}$	$1.60217653 \times 10^{-19}$	C	
$[p^+$, spin $(\mathbf{S})]_{-1,0}$	$9.132858671 \times 10^{-35}$	Js	$= 0.5\hbar\sqrt{3}$
$[p^+$, spin $(\mathbf{S_Z})]_{-1,0}$	$5.272858412 \times 10^{-35}$	Js	$= 0.5\hbar$
$[p^+$, surface area$]_{-1,0}$	$1.256637061 \times 10^{-29}$	m^2	
$[p^+$, volume$]_{-1,0}$	$4.188790205 \times 10^{-45}$	m^3	
$[p^+$, density$]_{-1,0}$	$3.993090196 \times 10^{17}$	kg/m^3	
$[p^+$, $q/M]_{-1,0}$	$9.578833758 \times 10^{7}$	C/kg	
$[p^+$, $\mu_p]_{-1,0}$	$1.410606712 \times 10^{-26}$	J/T	$= 2.792847351\ \mu_N$
$[p^+$, composition$]_{-1,0}$	88.636% [H_2 molecules]$_{-3,0}$ and 11.364% [He atoms]$_{-3,0}$ and 2.123×10^{40} [H_2^+]$_{-3,0}$		

9.3. Quantum scale neutron properties relative to the human scale

$[\text{n, radius}]_{-1,0}$	1.037057778	fm	
$[\text{n, mass}]_{-1,0}$	$1.67492728 \times 10^{-27}$	kg	
$[\text{n, charge}]_{-1,0}$	0		
$[\text{n, spin } (\mathbf{S})]_{-1,0}$	$9.132858671 \times 10^{-35}$	Js	$= 0.5\hbar\sqrt{3}$
$[\text{n, spin } (\mathbf{S_Z})]_{-1,0}$	$5.272858412 \times 10^{-35}$	Js	$= 0.5\hbar$
$[\text{n, surface area}]_{-1,0}$	$1.351499129 \times 10^{-29}$	m^2	
$[\text{n, volume}]_{-1,0}$	$4.671942228 \times 10^{-45}$	m^3	
$[\text{n, density}]_{-1,0}$	$3.585077037 \times 10^{17}$	kg/m^3	
$[\text{n}, \mu_S]_{-1,0}$	$-0.96623645 \times 10^{-26}$	J/T	$= -1.91304273\ \mu_N$
$[\text{n, composition}]_{-1,0}$	100.000% $[H_2 \text{ molecules}]_{-3,0}$		

9.4. Cosmic scale electron properties relative to the human scale

$[e^-, \text{radius}]_{1,0}$	$2.739673148 \times 10^{7}$	m
$[e^-, \text{mass}]_{1,0}$	$1.083591301 \times 10^{27}$	kg
$[e^-, \text{charge}]_{1,0}$	$3.401230\ 560 \times 10^{21}$	C
$[e^-, \text{spin } (\mathbf{S})]_{1,0}$	$4.115836850 \times 10^{46}$	Js
$[e^-, \text{spin } (\mathbf{S_Z})]_{1,0}$	$2.376279513 \times 10^{46}$	Js
$[e^-, \text{surface area}]_{1,0}$	$9.432077713 \times 10^{15}$	m^2
$[e^-, \text{volume}]_{1,0}$	$8.613603347 \times 10^{22}$	m^3
$[e^-, \text{mass density}]_{1,0}$	12,580	kg/m^3
$[e^-, q/M]_{1,0}$	$3.138850004 \times 10^{-6}$	C/kg
$[e^-, \mu_S]_{1,0}$	$-7.467434623 \times 10^{40}$	J/T
$[e^-, \sigma]_{1,0}$	360602.474	C/m^2
$[e^-, \text{composition}]_{1,0}$	excess of 2.123×10^{40} $[\text{electrons}]_{-1,0}$ on magnetic sphere of 1.169×10^{52} $[\text{Iron atoms}]_{-1,0}$	

9.5. Cosmic scale proton properties relative to the human scale

$[p^+, \text{radius}]_{1,0}$	$3.788565912 \times 10^{8}$	m
$[p^+, \text{mass}]_{1,0}$	$1.989639050 \times 10^{30}$	kg
$[p^+, \text{charge}]_{1,0}$	$3.401230560 \times 10^{21}$	C
$[p^+, \text{spin } (\mathbf{S})]_{1,0}$	$4.115836850 \times 10^{46}$	Js
$[p^+, \text{spin } (\mathbf{S_Z})]_{1,0}$	$2.376279513 \times 10^{46}$	Js
$[p^+, \text{surface area}]_{1,0}$	$1.803680287 \times 10^{18}$	m^2
$[p^+, \text{volume}]_{1,0}$	$2.277787217 \times 10^{26}$	m^3
$[p^+, \text{mass density}]_{1,0}$	8734.964509	kg/m^3
$[p^+, q/M]_{1,0}$	$1.709471153 \times 10^{-9}$	C/kg
$[p^+, \mu_p]_{1,0}$	$1.134505225 \times 10^{38}$	J/T
$[p^+, \sigma]_{1,0}$	1885.716989	C/m^2
$[p^+, \text{composition}]_{1,0}$	88.636 % $[H_2 \text{ molecules}]_{-1,0}$ and 11.364% $[\text{He atoms}]_{-1,0}$ and 2.123×10^{40} $[H_2^+]_{-1,0}$	

9.6. Cosmic scale neutron properties relative to the human scale

$[\text{n, mass}]_{1,0}$	$1.992381602 \times 10^{30}$	kg
$[\text{n, radius}]_{1,0}$	$3.928941005 \times 10^{8}$	m
$[\text{n, charge}]_{1,0}$	0	
$[\text{n, spin } (\mathbf{S})]_{1,0}$	$4.115836850 \times 10^{46}$	Js
$[\text{n, spin } (\mathbf{S_Z})]_{1,0}$	$2.376279513 \times 10^{46}$	Js
$[\text{n, surface area}]_{1,0}$	$1.939817529 \times 10^{18}$	m^2
$[\text{n, volume}]_{1,0}$	$2.540476211 \times 10^{26}$	m^3
$[\text{n, density}]_{1,0}$	7842.551697	kg/m^3
$[\text{n}, \mu_S]_{1,0}$	$-7.771126365 \times 10^{37}$	J/T
$[\text{n, composition}]_{1,0}$	100.000% $[H_2 \text{ molecules}]_{-1,0}$	

10. Observables of Electrons, Protons, and Neutrons in Several Scales

10.1. Human scale mass, radius, and charge of an electron located in seven scales (see Table 12a)

Table 12a. Electron located in seven scales with properties measured relative to the human scale

Electron in Scale m	Human scale $(n = 0)$ measures mass (kg)	Human scale $(n = 0)$ measures radius (m)	Human scale $(n = 0)$ measures charge (C)
5	1.53×10^{141}	3.9×10^{54}	1.53×10^{102}
3	1.29×10^{84}	1.0×10^{31}	7.22×10^{61}
1	1.08×10^{27}	2.7×10^{7}	3.40×10^{21}
–1	9.109×10^{-31}	7.23×10^{-17}	1.602×10^{-19}
–3	7.66×10^{-88}	1.9×10^{-40}	7.55×10^{-60}
–5	6.44×10^{-145}	5.0×10^{-64}	3.56×10^{-100}
–7	5.41×10^{-202}	1.3×10^{-87}	1.67×10^{-140}

10.2. Human scale mass, radius, and magnetic moment of a proton located in seven scales (see Table 12b).

Table 12b. Proton located in seven scales with properties measured relative to the human scale

Proton in Scale m	Human scale $(n = 0)$ measures mass (kg)	Human scale $(n = 0)$ measures radius (m)	Human scale $(n = 0)$ measures Magnetic moment, $\boldsymbol{\mu}$ (J/T)
5	2.82×10^{144}	5.4×10^{55}	7.34×10^{165}
3	2.37×10^{87}	1.4×10^{32}	9.12×10^{101}
1	1.99×10^{30}	3.8×10^{8}	1.13×10^{38}
–1	1.673×10^{-27}	1.00×10^{-15}	1.411×10^{-26}
–3	1.41×10^{-84}	2.6×10^{-39}	1.75×10^{-90}
–5	1.18×10^{-141}	7.0×10^{-63}	2.18×10^{-154}
–7	9.94×10^{-199}	1.8×10^{-86}	2.71×10^{-218}

10.3. Human scale mass and spin of a neutron located in seven scales (see Table 12c)**.**

Table 12c. Neutron located in seven scales measured at $n = 0$

Neutron in Scale m	Human scale $(n = 0)$ measures mass (kg)	Human scale $(n = 0)$ measures spin, $\mathbf{S_Z}$ (Js)
5	2.82×10^{144}	4.83×10^{207}
3	2.37×10^{87}	1.07×10^{127}
1	1.99×10^{30}	2.38×10^{46}
–1	1.675×10^{-27}	5.273×10^{-35}
–3	1.41×10^{-84}	1.17×10^{-115}
–5	1.18×10^{-141}	2.60×10^{-196}
–7	9.95×10^{-199}	5.76×10^{-277}

11. Scaling Fractal Discussion

11.1. String theory and subquantum scale atoms

In string theory particles are perceived as highly localized vibration of Planck length strings.

$$l_p = (\hbar_{-1,0} G_{1,0} c^{-3})^{1/2}$$

$$= [(1.0545717 \times 10^{-34}\,\text{Js})(6.6742 \times 10^{-11}\,\text{Nm}^2/\text{kg}^2)(299792458\ \text{m/s})^{-3}]^{1/2}$$

$$= 1.62 \times 10^{-35}\,\text{m}.$$

From the Bohr model of the Hydrogen atom and in particular quantized angular momentum:

$$m v_n r_n = n\hbar.$$

Combined with the de Broglie relation:

$$\lambda = h/p.$$

The relation has long been known:

$$2\pi r_n = nh/p_n = n\lambda.$$

The smallest atomic orbital circumferences are the ground state Helium shells ($1s^2$ orbital) of the heaviest atoms. The diameter of the helium shell for Radon $(z = 86) \sim 0.02\text{Å}$. This shell has an average circumference $= (2\pi)(1 \times 10^{-12}\,\text{m}) = 6.28 \times 10^{-12}\,\text{m}$, which is also the smallest ground state wavelength of an atomic orbital in the human scale.

Fractal Physics Theory length scaling fractal:

$$¥L = 3.789 \times 10^{23}.$$

A self-similar Radon atom existing at the subquantum scale will have a self-similar subquantum scale $1s^2$ orbital circumference measured relative to the human scale:

$$[\text{Radon, } 1s^2 \text{ circumferenece}]_{-3,0} = 6.28 \times 10^{-12}\,\text{m}/3.789 \times 10^{23} = 1.66 \times 10^{-35}\,\text{m}.$$

Vibrating string particles correspond to subquantum scale atoms. Strings are subquantum scale atomic oscillators.

11.2. Unified fields of scale

The unified field equation of Fractal Physics:

$$¥\mathbf{B}¥\mathbf{E}¥\mathbf{g} = ¥M/¥L^4. \tag{19}$$

The product of the three basic fields scale as the density of space-time scales.

The gravitational field scaling fractal equals the linear acceleration scaling fractal: $¥\mathbf{g} = ¥\mathbf{a} = ¥L^{-1}$. This equality is similar to the principle of equivalence. The gravitational field scaling fractal also equals the surface area to volume ratio scaling fractal: $¥\mathbf{g} = ¥\text{SA}/\text{Vol} = ¥L^{-1}$. This is remarkable in that the gravitational field scales without any, a priori, requirement of particle masses. The product of the electric and magnetic fields scale as density or temperature or pressure scales:

$$¥\mathbf{B}¥\mathbf{E} = ¥D = ¥T = ¥Pr. \tag{20}$$

This is also remarkable in that density, temperature, and pressure are all understood with an a priori requirement of particle masses.

11.3. Fractal certainty principle

Planck's constant is not scale invariant. Consider the Heisenberg uncertainty principle:

$$\Delta x \Delta p \geq 0.5\hbar, \qquad \Delta t \Delta E \geq 0.5\hbar$$

Measuring an electron's position with subquantum scale electromagnetic radiation (a small percent of a single neutrino) will not significantly alter the electron's momentum. The position and momentum of an electron can be determined, for practical purposes, simultaneously.

The Fractal certainty principle:

$$[\Delta x]_{m,n}[\Delta p]_{m,n} \geq [0.5\hbar]_{m,n} \qquad [\Delta t]_{m,n}[\Delta E]_{m,n} \geq [0.5\hbar]_{m,n} \tag{21}$$

The Fractal certainty principle embodies a paradigm shift for scientific inquiry. Fractal Physics Theory assumes that it is always possible for humans to draw a picture of any physical phenomena and that the picture can truly reflect the reality of the phenomena. Furthermore, FPT assumes that based on the accurate visual representation of the phenomena, physical equations can be determined and written to detail the specific cause and effect of the physical phenomena, accurately, realistically, and completely.

11.4. Planck values

Modern Physics Planck values do not compare to common Earth science limits (Table 13a). The mass is very far above atomic masses, the temperature is very far above fusion temperatures, the length is very far below nuclear dimensions and the time is very far below elementary particle half-lives.

Table 13a. Planck values in modern physics $(G = 6.6742 \times 10^{-11}\,\mathrm{Nm}^2/\mathrm{kg}^2)$ [15].

Planck		Formula	Value	Units
Mass	m_p	$(\hbar c/G)^{1/2}$	2.176451×10^{-8}	kg
Temperature	T_p	$(\hbar c^5/G)^{1/2}/k$	1.416792×10^{32}	K
Length	l_p	$(\hbar G/c^3)^{1/2}$	1.616243×10^{-35}	m
Time	t_p	$(\hbar G/c^5)^{1/2}$	5.391206×10^{-44}	s

Fractal Physics Planck values compare well to known Earth science limits (Table 13b). The mass is 234 amu, about the limit of nuclear masses, the temperature is somewhat beyond high fusion temperatures, the length is 1% of the electron's diameter, and the time is ppm fraction of short lived elementary particle half-lives like Σ^0.

Table 13b. Planck Values in Fractal Physics ($G_{-1,0} = 2.095564 \times 10^{23}\,\text{Nm}^2/\text{kg}^2$)

Planck		Formula	Value	Units
Mass	m_p	$(\hbar c/G)^{1/2}$	3.884167×10^{-25}	kg
Temperature	T_p	$(\hbar c^5/G)^{1/2}/k$	2.528456×10^{15}	K
Length	l_p	$(\hbar G/c^3)^{1/2}$	9.056439×10^{-19}	m
Time	t_p	$(\hbar G/c^5)^{1/2}$	3.020903×10^{-27}	s

11.5. Scalativity illustrated by a mole of iron

A mole of iron atoms can be used to depict Scalativity.

Let iron's atomic radius $= 1.2 \times 10^{-10}\,\text{m}$.

A $55.8\,\text{g}$ iron sphere is a human scale object relative to the human scale:

$[\text{iron sphere, mass}]_{0,0} = 55.8\,\text{g} = 1\,\text{mol}$,

$[\text{iron sphere, density}]_{0,0} = 7800\,\text{kg/m}^3$,

$[\text{iron sphere, radius}]_{0,0} = 1.2\,\text{cm}$.

One iron atom of this sphere is a quantum scale object relative to the human scale:

$[\text{iron atom, mass}]_{-1,0} = 9.27 \times 10^{-26}\,\text{kg}$,

$[\text{iron atom, diameter}]_{-1,0} = 2.40 \times 10^{-10}\,\text{m}$.

This same iron atom is a cosmic scale object relative to the lilliputian scale:

$[\text{iron atom, mass}]_{-1,-2} = 1.10 \times 10^{32}\,\text{kg} = 55.4$ solar masses,

$[\text{iron atom, diameter}]_{-1,-2} = 9.09 \times 10^{13}\,\text{m} = 608$ astronomical units.

Take each atom of this iron sphere, 6.0221415×10^{23} atoms, and place them in a "solid" straight line, aligned with each atom touching. This line of iron atoms is a cosmic scale object relative to the human scale:

$[\text{line of iron atoms, length}]_{1,0} = 1.45 \times 10^{14}\,\text{m} = 969$ astronomical units.

This same iron atom line is a quantum scale object relative to the titanic scale:

$[\text{line of iron atoms, length}]_{1,2} = 3.83 \times 10^{-10}\,\text{m}$.

12. Conclusion

This article discusses the foundation of Fractal Physics Theory, establishes profound connections to E-infinity Theory, tabulates many scaling fractals and constants, which are derived by identifying the solar system as a cosmic scale neutron midway through cosmic scale beta decay. Fractal Physics Theory introduces a unified field equation of scale, the Fractal Certainty Principal and strings as subquantum scale atomic orbitals.

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