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GROUP VELOCITY OF LIGHT IN REALITY: PROPERTIES AND APPLICATIONS

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Abstract

By definition, group velocity of light (GV) in vacuum is the velocity of a wave packet (WP) which is often named as pulse. GV is generally accepted to be invariant to the reference frame. However, it is simply shown in this article that WP moves in Galilean space-time at non-relativistic velocities of the observer as well as particles in standard quantum mechanics. General opinion is based on exact astronomical observations of natural light of binary stars (BS). It is shown in the article that WP does not exist in usual natural light because of large eikonal value. Thus, results with BS are explained and contradiction is solved. Therefore, WP can take place only at a small eikonal value, that is, for short radar, laser pulses, and for field from the spontaneous emission of a single atom.

1. Introduction

We will confine our attention to light velocities in vacuum. Classical optics asserts that there are two kinds of light velocity: phase velocity (PV) and group velocity (GV) (see [1, 2]). It is well known that PV is invariant to the reference frame (RF) of observer [3].

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The situation with group velocity (GV) is not so simple. The overwhelming majority of specialists on classical optics are convinced that GV and PV are equal in vacuum [1, 2, 3]. This opinion was advanced in [4] and refuted in Section 3 of this article.

It is worth noting that just light pulse non-invariance was never tested seriously in an experiment using radar or laser pulses.

Only experiment in [5] is precise enough for serious discussion. But it is shown in Section 4 that authors [5] actually measured PV but not GV.

There is published summary of numerous radar ranging data for Mercury [6], and it is easy to estimate that this data is sufficiently precise for testing the invariance of radar pulse velocity. Unfortunately, only professional astronomers can process this data accurately taking into account heights of mountains on Mercury.

Unfortunately, nobody tried to calculate strictly the dependence of exact GV of light pulse on the RF. In well known textbooks [1, 2], authors constructed their wave packets for theoretical consideration merely from two plane waves. It is indecently. Further, they introduced crude approximations into these two waves and got wishful invariance of GV to RF.

Therefore, a precise theoretical investigation of the properties of GV is the goal of this article. The remainder of the paper is organized as follows: Section 2 contains definitions and analyses of the simplest properties of GV by standard wave-packet method. Natural light invariance to RF is disproved in Section 3. It is shown in Section 4 that light pulses are created by synchrotron as a whole, but not by separate electrons inside it. Section 5 offers a short conclusion.

2. Wave Packets of Light and their Properties

The usual definition of GV is considered to be the velocity with which the overall shape (envelope) of the group of waves propagates as a whole through space [1, 2]. Such groups of waves are usually called wave packets (WP) - not to be confused with photons. It is worth noting that the existence of WP is a strict consequence of the superposition principle for light, but not a result of a special postulate.

WP have been well-examined in quantum mechanics for waves of particles in different RFs (see [7]). It is well known that such WP, with non-relativistic velocities, moves in Galilean space-time. Yet the description of WP of light (WPL) is considerably simpler than for particle waves because photon mass is usually accepted to be null. Nevertheless, WPL motion was never specifically considered with different RFs. Below, we will do this simple work.

We will construct the simplest WPL from the phase factor of a plane wave, which can be written as $\exp(i(kz - \omega t))$ in atomic units. This plane wave propagates in the direction of z, c is PV, and $ck = \omega$. We then get S which is amplitude of WPL, which is radiated by an immobile light source:

$$S = \int_{-\infty}^{\infty} \exp\left(i\omega\left(\frac{z - ct}{c}\right)\right) f(\omega - \omega_0) d\omega$$

$$= 2\pi \exp(i\omega_0 \eta) F(\eta), \qquad (1)$$

where $f(\omega - \omega_0)$ is some real weight function of frequency, $F(\eta)$ is its Fourier transform over variable $(\omega - \omega_0)$, $\eta = \frac{z - ct}{c}$ and $\psi = \omega_0 \eta$ can be considered as eikonal.

We have neglected the properties of polarization vector, because small changes in it introduce into S additions with small amplitudes.

Besides this exception, equation (1) is a strict and general, as well as standard method of Fourier transformations.

Let us describe the mathematical mechanism of motion for WPL in detail. There exist two simple conditions: First, function $F(\eta)$ depends only on η but not on variables z and t separately; second, there exists a flow of time, which is independent of us. Both conditions are satisfied if WPL moves in space and time as a whole without any changes. If we watch some definite value of quantity η, we will see motion of WPL as a whole with velocity V. It is obvious that our WPL moves with velocity $V = c$ as a whole, and likewise with wave packets in quantum mechanics (e.g., [7]) - but without longitudinal spreading with time.

Having looked at some handbooks on Fourier transformations, we can get simple estimations for $F(\eta)$, which has the following usual properties. It decreases as the function of variable η, if $|\eta|$ is large enough; besides, $F(\eta)$ is usually a smooth function and has a maximum near $\eta \approx 0$ if the weight function $f(\omega - \omega_0)$ is a smooth one. If not, $F(\eta)$ is usually not smooth either. The simplest example: if $f(\omega) = \exp(-\alpha\omega^2)$ and $\alpha > 0$, we get

$$F(\eta) = \sqrt{\frac{\pi}{\alpha}} \exp\left(-\frac{\eta^2}{4\alpha}\right). \tag{2}$$

There exists another simple and real example of WPL: the electromagnetic field of spontaneous emission of an immobile atom, which represents WPL in the analytical form of limited spherical divergent wave [8].

Equation (1) describes the motion of WPL as light source, which is immobile relative to the observer. In this case, WPL moves in space in the direction of z with the velocity equal to constant c, that is, in this case GV and PV coincide. This result also coincides with the simplest result of classical optics, which was usually calculated for both the immobile light source and the observer. We can say that c is the proper velocity of WPL. We will use below unit vector $\mathbf{n}$ in the line of z.

If the observer has a constant velocity $\mathbf{v}$ in RF of the light source, it is obvious that observing the projection of velocity of WPL in the direction of $\mathbf{n}$ that GV of our WPL is

$$V = (c - \mathbf{nv}). \tag{3}$$

It is evident from equation (3) that the modulus of WPL moves in Galilean space-time similar to that of the WP of particles in non-relativistic quantum mechanics. At that, we have not introduced any hypotheses into standard description of light.

3. Wave Packets of Light and Eikonal Value

It is obvious that real natural light is always a result of radiation from numerous sources, which are in turn excited in a very complicated way. Therefore, in general, natural light represents a superposition of different and very complex variants of WPL radiated by different atoms of the source. These atoms were excited in different ways. But GV of every single WPL obeys equation (3) therefore GV is non-invariant to RF. If so, the results of observing of moving sources radiating natural light would be strongly distorted [4]. However, it is not so: astronomers see the exact picture of motion of binary stars. Therefore, it was decided simply that GV as well as PV is invariant in general [4].

But it was not noticed that this conclusion contradicts the trivial properties of light, which were used in elementary derivation of equations (1) for WPL. This situation turned out to be interesting enough.

The fact is that astronomers use methods of geometric optics (GO), which represents well-known simplification of natural light. We know that optical telescopes based on GO are widely applicable in astronomy including observations of binary stars. But GO requires the value of eikonal ψ to be very large [2, 9], whereas we know (see equation (2)) that in general, $F(\eta) \to 0$ if $\psi = \omega_0 \eta \to \infty$.

This implies that practically, WPLs do not exist in that natural light, which can be transmitted without distortions through usual optical instruments. So, it is possible to say that light with GV does not exist if we can apply GO. Therefore, we see that the speed of light, emitted by binary stars, have to be invariant to RF as well as to PV. So, astronomers see the exact picture of binary star motion without distortion, which would be introduced by the non-invariant velocity of WPL. This implies that actually in nature there are no contradictions with equations (1). Thus, we have refuted mentioned objection against GV non-invariance, in principle.

Therefore, the only opportunity for WPL existing in pure form takes place with short radar, laser pulses, and spontaneous emission of a single atom, excited by short pulses. In all these cases, light is created by a simple source with a single “resonator”. It is worth noting that the first two “resonators” are macroscopic.

4. On the Role of Different Parts of Light Sources

It was shown in a good recent experiment [5] that the velocity of light pulses, radiated by synchrotron, does not depend on the corresponding velocity projection of radiating electrons. The authors of [5] interpreted this result as a proof of light velocity invariance, having made no mention of the possible difference between PV and GV. Their conclusion obviously contradicts the equation (3). I will explain their result using equation (1), properties of synchrotron radiation, and results of the study [8].

We begin from the obvious consequences of requiring the wholeness of WPL. This requirement implies that the concept of GV has physical meaning only for a single WPL, described by equation (1), as a whole but not for any of its parts.

Further, it follows from equations (1), (2), and (3) that the velocity of propagation of WPL does not depend on the large velocities of separate radiating electrons of atoms. This implies that radiating electrons are mere “details” within the radiating immobile source. We can therefore conclude that we should consider velocity $\mathbf{v}$ in equation (3) as the velocity of the motion of the “resonator” of the light source relatively observer, but not as the velocity of radiating electrons in synchrotron. This conclusion is in agreement with strict formulas for the field of spontaneous emission of atoms in [8]. These formulas do not contain velocities of electrons of the emitting atom.

We will comprehend more deeply if we consider the general formulas for Lienard-Wichert potential, which represent the foundation of the synchrotron radiation theory. In a derivation of these formulas, we mean that the source of the magnetic field is immobile and radiating light velocity is equal to c. These formulas do not contain expressions such as $(\mathbf{n}\mathbf{v}_e + c)t$, where $\mathbf{v}_e$ is the velocity of the radiating particle in RF of observer (see [9, 10]). This implies that the velocity of the pulses of synchrotron radiation does not depend on the projections of velocity of radiating electrons. Just such a result was obtained in experiment [5] because synchrotron was a light source, which as a whole, was immobile relatively observer.

5. Conclusion

PV is actually a number parameter of the theory of electromagnetic processes and relativity theory. This parameter has velocity dimension. Apparently, it is the only physical meaning of PV.

GV is the velocity of a WPL as a whole by definition. A single WPL moves in Galilean space-time at non-relativistic velocities of the light source relative to the observer. This result agrees with the motion of WP in quantum mechanics. As such, we should consider the light source as a whole “resonator” without details if we are limited by light pulse motion. Astronomers see the exact picture of binary stars because WPLs and their GV practically do not exist in rays of natural light if we can apply GO to these rays. This situation arises because of the large value of eikonal needed for the application of GO. Only the motion of short radar, laser pulses, and WPL of the spontaneous emission of a single atom represents the motion of WPL with GV. The usual ray of natural light moves with PV, which is equal to c.

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